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TITLE: G.fast: Improved model for shunt admittance in G.fast cable model

ABSTRACT

This contribution discusses limitations with the provisionally agreed G.fast cable model, specifically in terms of modeling shunt admittance for cables with high-loss dielectrics (e.g. PVC). The limitation of the existing model is due to the fact that it has only two parameters to model the admittance while it has four parameters to model the series impedance. The contribution also indicates a possible improvement of the model that addresses the mentioned issues. This contribution is for information only.

1. Introduction

A transmission line can be represented as a distributed model using an infinite series of cascaded two-port networks. Ideally, these networks are all identical but for practical transmission lines there would be impedance variations due to manufacturing imperfections etc. Furthermore, each two-port corresponds to an infinitesimally short segment *dx* of the total line length. This forms the basis for the well known “Telegrapher’s Equations” and is illustrated in Figure 1 below. The two-port can be divided into a series impedance part composed of resistance *R* and inductance *L*, as well as a shunt admittance part composed of conductance *G* and capacitance *C*

$$Z_s(jw)dx = \left(R(w) + jwL(w)\right)dx$$
$$Y_p(jw)dx = \left(G(w) + jwC(w)\right)dx\,.$$

This contribution focuses on the shunt admittance part. Further, a commonly used definition is the so called “loss tangent” that describes the relationship between conductance and capacitance

$$\tan d = \frac{\mathrm{Re}(Y_p)}{\mathrm{Im}(Y_p)} = \frac{G}{wC}$$

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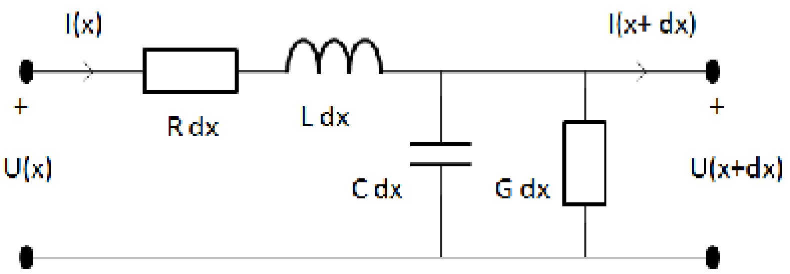

Figure 1 Electrical model of a segment from a twisted pair copper cable.

Since R, L, C, and G may all be frequency dependent, a suitable model is needed to describe the proper behavior. Various models are proposed in the literature and perhaps the most common model used for xDSL purposes is the BT0 model, [1][2], which says that

$$R(w) = \sqrt[4]{r_{0c}^{\ 4} + a_c f^2}$$

$$L(w) = \frac{l_0 + l_\infty \left(\frac{f}{f_m}\right)^b}{1 + l_\infty \left(\frac{f}{f_m}\right)^b}$$

$$G(w) = g_0 f^{g_e}$$

$$C(w) = c_\infty + c_0 f^{-c_e}$$

where $f = w/2p$.

The BT0 model has been used successfully to describe many different types of cables with different geometries, conductor thicknesses, and dielectric materials. However, one limitation is the lack of causality, which means that it is not suitable for time-domain simulations. Furthermore, it has been seen that when using the BT0 parameter sets defined for xDSL bands, it is not always accurate to extrapolate above 30 MHz, as is required for G.fast applications. It has therefore been proposed [3] and agreed to define new wideband models covering the G.fast frequency range. Such models were proposed by TNO [4][5] where the series impedance and shunt admittance is modeled as

$$Z_s(jw) = jwL_{s\infty} + R_{s0}\left(1 - q_s + q_s Q_Z\left(\frac{jw}{q^2 w_s}\right)\right)$$

$$Y_p(jw) = jwC_{p0}\left[Q_Y\left(\frac{jw}{w_d}\right)\right]^f$$

where $Q_Z(jw)$ and $Q_Y(jw)$ are analytical shaping functions with the following properties

$$Q_Z(jw) \to 1 + jw \qquad w \to 0$$

$$Q_Z(jw) \to (1+j)\sqrt{w} \qquad w \to \infty$$

$$Q_Z(jw) = conj(Q_Z(-jw)) \qquad \forall w$$

and

$$Q_Y(jw) \to 1 \qquad w \to 0$$

$$|Q_Y(jw)| < 1 \qquad w \to \infty$$

$$\arg(Q_Y(jw)) \to 1 \text{ over several decades} \qquad w \to \infty$$

$$Q_Y(jw) = conj(Q_Y(-jw)) \qquad \forall w$$

In the document [4] several shaping functions has been described, referring back to other models. One set of functions is

$$Q_Z(jw) = \sqrt{1 + j2w}$$

$$Q_Y(jw) = \left(\frac{1}{1 + jw}\right)^{\frac{2}{p}}$$

This leads to

$$Z_s(jw) = jwL_{s\infty} + R_{s0}\left(1 - q_s + \sqrt{q_s^{\,2} + 2\frac{jw}{w_s}}\right)$$

$$Y_p(jw) = jwC_{p0}\left(\frac{1}{1 + jw/w_d}\right)^{\frac{2f}{p}}$$

where the latter is the model for shunt admittance, which is the focus of improvement in this contribution.

2. Problem description

A limitation of the TNO model for shunt admittance is that it can only accurately model cables with constant loss tangent. This limitation is expected since the model has only two parameters, C_{p0}, f, and thus cannot be expected to describe complicated frequency dependent behavior (for comparison, the series impedance has four or more parameters). The constant loss tangent approximation may be acceptable for low-loss cables but is not good enough for e.g. polyvinylchloride (PVC) insulation, which may be found in drop cables and some indoor telephone cables. Even for the low-loss polyethylene (PE) insulation, the constant loss tangent assumption may be inaccurate if the frequency is high enough.

Ref. [6] describes dielectric properties for various polymers and it can be seen that PVC at room temperature has a peak in the loss tangent at a few tens of kHz and thereafter decreases with frequency, at least up to about 10 GHz where it starts to increase again. PE on the other hand has a rather constant loss tangent up to 1 – 10 MHz (depending on density), which then starts to increase and may have doubled or tripled at a few hundred MHz compared to the low-frequency value. In order to improve accuracy, a model is needed where the loss tangent can be described as a slope (in log-log scale) versus frequency.

3. Possible solution

The conductance and the capacitance of the TNO model can be derived as the real and imaginary part of the admittance, respectively. By choosing $w_d << w$ for all relevant frequencies, the admittance can be rewritten using suitable identities, including Euler's formula

$$Y_p(jw) = C_{p0}w_d^{\frac{2f}{p}}\sin(f)w^{1-\frac{2f}{p}} + jwC_{p0}w_d^{\frac{2f}{p}}\cos(f)w^{-\frac{2f}{p}}$$

While this looks complicated at first, it allows the conductance to be written in a similar way as in the BT0 model, i.e. $G(w) = g_0 f^{g_e}$ $G(j\omega) = g_f f \omega^{g_e}$, where $f = w/2p$, by identifying

$$g_e = 1 - \frac{2f}{p}$$

$$g_0 = 2pC_{p0}\left(\frac{w_d}{2p}\right)^{1-g_e}\sin(f)$$

The exponent, g_e, is normally close to unity due to fact that f is determined by the second equation above in order to make the magnitude of $G(w)$ fit measurements. Then f is small, which means $g_e \approx 1$ and $g_0 \approx 2pC_{p0}f$. Actually, f can be interpreted as the slope of $G(w)$.

In the BT0 model, capacitance is modeled as $C(w) = c_\infty + c_0 f^{c_e}$ $C(j\omega) = C_\infty + C_0\omega^{C_e}$, but the additive constant

c_∞ is not included in the TNO model. This leads to that the loss tangent for the TNO model is constant since the conductance and the capacitive reactance have the same slope versus frequency,

$$\frac{G(w)}{wC(w)} = \tan(f)$$

As stated earlier, for PVC insulated cables the loss tangent is typically a decreasing function, while for PE insulated cables it is an increasing function [6] with frequency. Thus, it is necessary to include a sloping loss tangent in the model. To get the possibility to adapt the conductance to another slope than unity and still be able to adapt to the magnitude of both $G(w)$ and $C(w)$, an additional degree of freedom is required. Therefore, it is proposed to modify the expression for the admittance to

$$Y_p\left(jw\right) = jwC_{p0}\left(\left[Q_Y\left(\frac{jw}{w_d}\right)\right]^f (1-q_c) + q_c\right)$$

or, equivalently,

$$Y_p\left(jw\right) = jwC_{p0}(1-q_c)\left[Q_Y\left(\frac{jw}{w_d}\right)\right]^f + jwC_{p0}q_c$$

where q_c is a scaling parameter. Expressing Y_p via this third scaling parameter, q_c, introduces the required additional degree of freedom and also has the advantage that q_c can be viewed as for refining purposes. If $q_c = 0$, we get the original TNO model. If $q_c \neq 0$ (normally $q_c > 0$) it can match cables where the dielectric losses cannot be ignored. For $f > 0$,

$$q_c = \frac{C_{p0}}{C(w)_{w\to\infty}}$$

It should be noted that $f < 0$ is also possible and allows modeling of a loss tangent that increases with frequency but in that case, the interpretation of q_c above does not hold.

Compared to the original model, [4], the here described modification allows the value of f to be set more independently and is now reflecting the slopes of $G(w)$ and $C(w)$. Intuitively, the modification can be justified by noting that the original model has only two degrees of freedom, while there are three desired adjustments to measurements; e.g. the value at low frequencies, the value at high frequencies, and the slope. The modified model is able to cope with these requirements in the same way as the BT0 model.

4. The suggested model as a causal version of the BT0 admittance

An additional advantage of the modified model is that it can be interpreted as a causal version of the BT0 admittance. $C_1 + C_1 = C_{p0}$ Using the same shaping function as above, including the new scaling parameter q_c, we can write the admittance as

$$Y_p(jw) = C_{p0}(1-q_c)w_d^{\frac{2f}{p}}\sin(f)w^{1-\frac{2f}{p}} + jw\left(C_{p0}(1-q_c)w_d^{\frac{2f}{p}}\cos(f)w^{-\frac{2f}{p}} + C_{p0}q_c\right)$$

For $w >> w_d$, we can identify the suggested version with BT0 as will be shown in the following.

The conductance can be written as $G(w) = g_0 f^{g_e}$, where $f = w/2p$ and

$$g_e = 1 - \frac{2f}{p}$$

$$g_0 = 2pC_{p0}(1-q_c)\left(\frac{w_d}{2p}\right)^{1-g_e} \sin(f)$$

and the capacitance as $C(w) = c_0 f^{-c_e} + c_\infty$, where

$$c_e = \frac{2f}{p}$$

$$c_0 = C_{p0}(1-q_c)\left(\frac{w_d}{2p}\right)^{1-g_e} \cos(f)$$

$$c_\infty = C_{p0} q_c$$

Thus the modified model is within the class of the BT0 model. In essence, the modified model differs from the suggested model in [4][5] by a constant $C_{p0}q_c$ in $C(w)$, which does not affect the causality of the model. It has been seen by engineering experience that the suggested model does not violate the causality condition due to its construction, and hence, this also yields for the modified model. Hence, the mapping to the BT0 model gives a causal version of the BT0 admittance. The above also implies that the BT0 model can be made causal by letting its parameters fulfill the two relations

$$c_e + g_e = 1$$

$$g_0 = c_0 2p \tan\left(\frac{p}{2} c_e\right)$$

with the consequence that the number of parameters is reduced also by two, i.e. the number of equations to be fulfilled. Thus these two equations can be seen as causality conditions for the original BT0 model. It can also be seen that when identifying with BT0, the five parameters, c_0, c_e, c_∞, g_0 and g_e, can be expressed in terms of only three essential parameters, f, C_{p0} and q_c.

5. Numerical Results

In **Figure 2**, capacitance and conductance are shown for a 1.00 meter PVC-insulated indoor telephone cable (2 twisted pairs). The blue curves are the measured values, which are derived from input impedance measurements on open/short termination as in [2]. Here, measured values are inaccurate above 10 MHz. Three model results are shown: Green represents the TNO admittance model fitted to measurements around 1.5 MHz, while red represents the same model fitted to extrapolated measurement data around 60 MHz, and finally, cyan is the modified admittance model proposed in the current text. We see that the existing model has limitations when applied to PVC-insulated cables; for capacitance, it can only model the slope in a narrow bandwidth and for conductance $G(w)/w$ only the level at one specific frequency can be modeled. In **Figure 3** the loss tangent, $G(w)/wC(w)$, for the same measurement and models are shown. It can be seen here that the loss tangent for the TNO model is constant (as mentioned above), while for the modified model it can be tailored to follow the measurements in a better way.

In **Figure 4**, measured insertion loss (IL) is shown for a 24.1 meter cable of the same type, together with predicted IL from the admittance models discussed above. The measured data looks noisy above 100 MHz, probably due to the fact that this is a low-quality cable, which is not designed for high frequencies. In order to study effects of admittance errors, all three models use the series impedance Zs from the BT0-Hilbert model [7]. Any errors in the series impedance model will then be identical for the three cases. A further source of error is that predictions are based on a 1.0 meter segment while the insertion loss measurements are taken on a 24.1 meter segment from the same cable. Small differences in cable properties are expected between the two cables but again, the error will be the same for all compared models. It can be seen in **Figure 4** that the TNO model fitted at 1.5 MHz overestimates IL at high frequencies, mainly due to the error in $G(w)/w$. Here, the TNO model fitted at 60 MHz has a visible but small absolute deviation, at least up to 100 MHz while the modified model follows the measured data very well for all frequencies.

Figure 5 shows that although the absolute deviation was small for the TNO model fitted at 60 MHz, the relative deviation is still large for frequencies sufficiently above or below the fitting frequency. For example, the relative dB error below 20 MHz is more than 10%. The modified model shows a much smaller relative deviation (below 5% relative deviation for almost all frequencies up to 200 MHz), which could be even further improved by tuning the model to the loss of the 24.1 meter segment instead of calculating loss based on model fitting to the 1.0 meter cable. The existing model would only gain marginally from such tuning since it cannot give correct shape for $C(w)$ and

$G(w)/w$. The results presented above show that at least for PVC-insulated cables, the modified model better describes the cable behavior than the current ITU-T G.fast cable model.

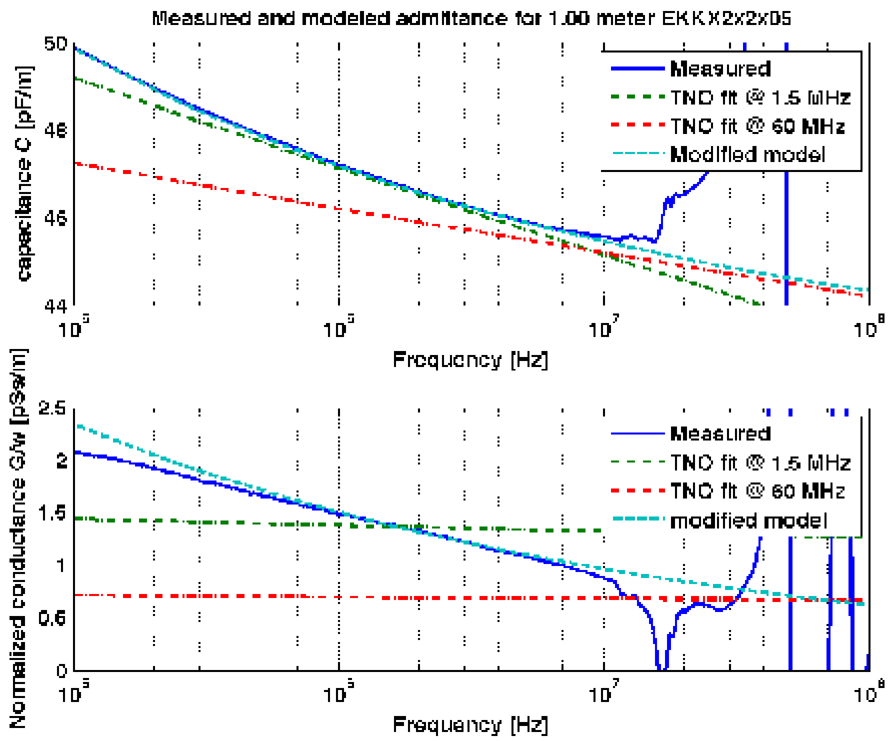

Figure 2 C and G measurement and models for a PVC-insulated indoor telephone cable. The first TNO model (green) is a fit to measured data around 1.5 MHz and the second TNO model (red) is a fit to extrapolated measurement data around 60 MHz. The modified model (cyan) is fitted around 1.5 MHz. Measurement data is inaccurate above 5 – 10 MHz.

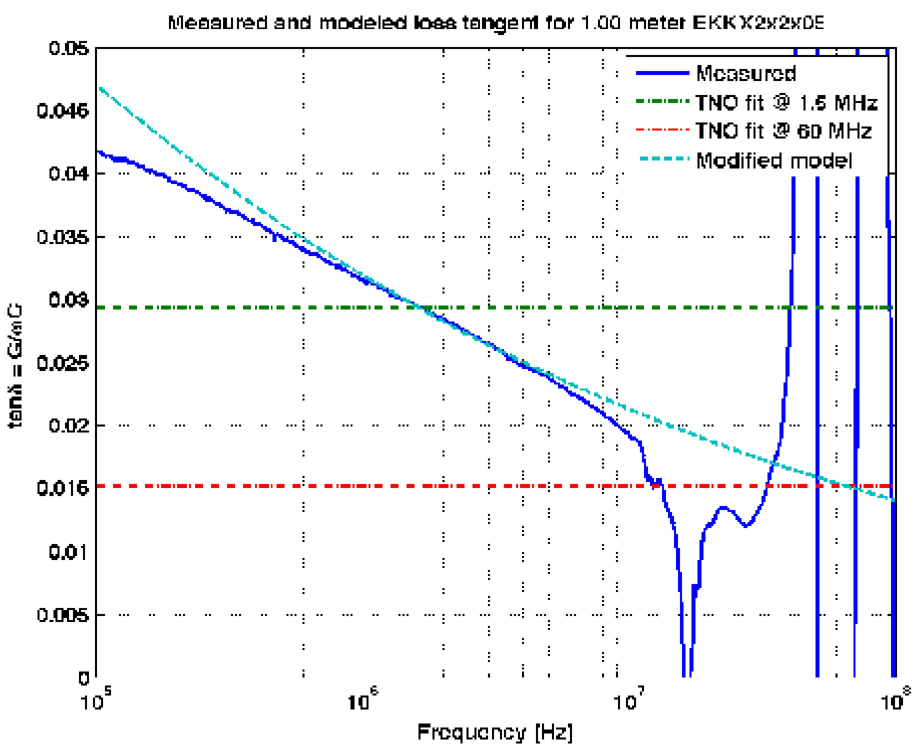

Figure 3 Loss tangent measurement and models for a PVC-insulated indoor telephone cable. The TNO models, fit to measured data around 1.5 MHz and 60 MHz, respectively, are constant, while the modified model follow the measurements better. Measurement data is inaccurate above 5 – 10 MHz.

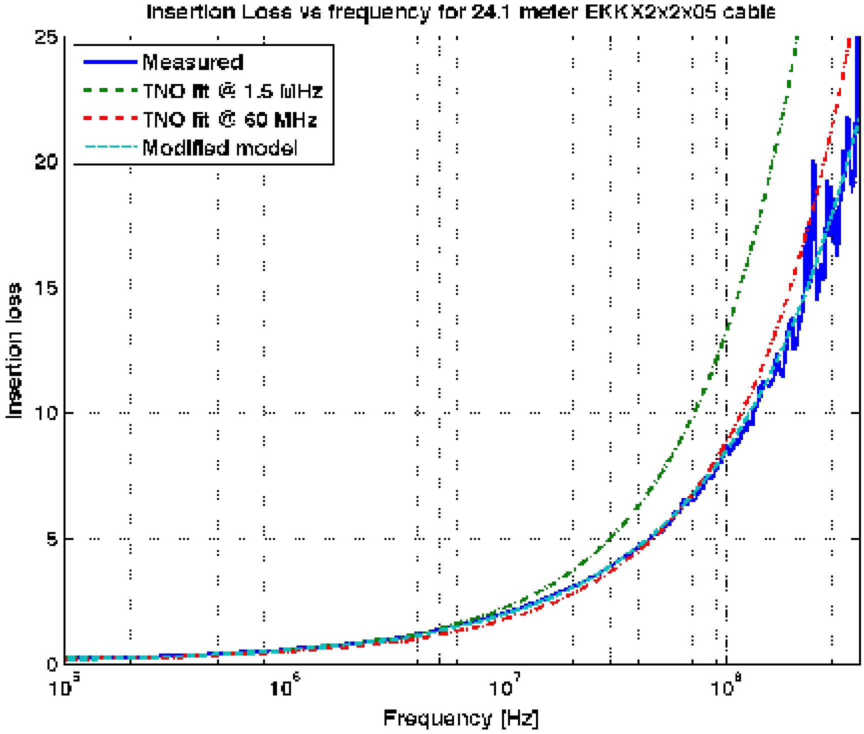

Figure 4 Insertion loss measurement and models for a 24.1 meter long PVC-insulated indoor telephone cable.

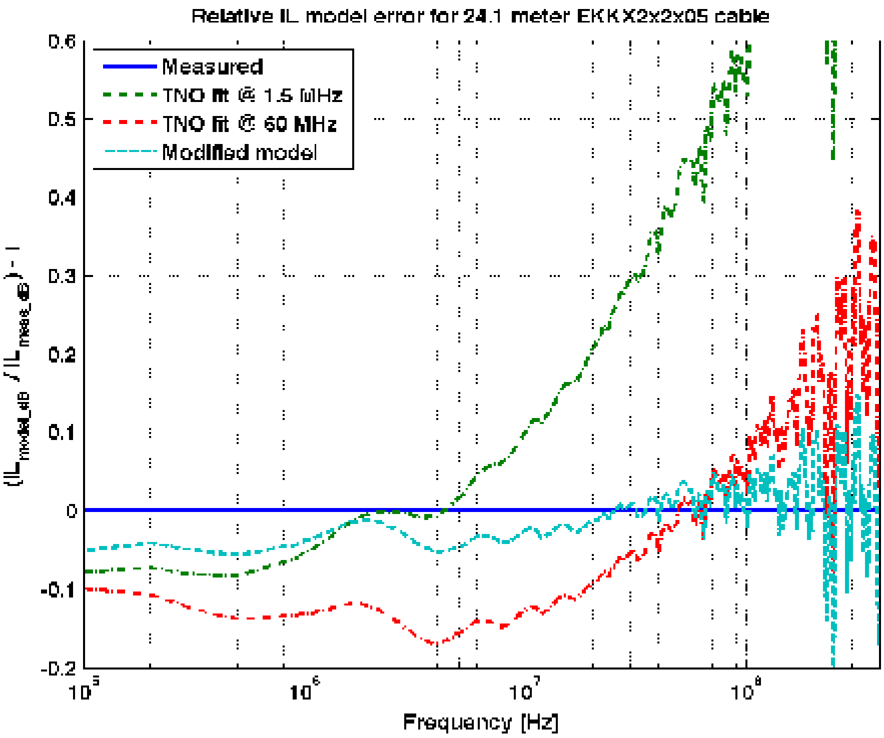

Figure 5 Relative error between insertion loss calculated from models and an insertion loss measurement for a 24.1 meter long PVC-insulated indoor telephone cable

6. Summary

This contribution should be presented under G.fast. The contribution describes issues related to cable models for G.fast with focus on the shunt admittance (conductance and capacitance). A modification of the current model is shown, substantially improving model accuracy for PVC insulated cables while still giving backward compatibility with the TNO model. A further advantage is that parameter interpretations can be reused from the BT0 model.

7. Acknowledgements

Ericsson acknowledges the contributions from Prof. Per Ola Börjesson and Assoc. Prof. Stefan Höst from Lund University. They made a substantial effort especially regarding derivations of the analytical formulas for the cable models.

8. References

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