
TITLE **Generalizing the crosstalk sum in a multi-impedance environment**

PROJECTS ADSL

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ABSTRACT Summing crosstalk, originating from different sources with different impedance is an issue that is not fully understood. The application of the well known FSAN crosstalk sum to this more general case has resulted in different interpretations on how to perform this generalized summation. Differences up to 1.3 dB can be observed when evaluating the noise models for ADSL. This contribution discusses the most plausible way to perform crosstalk summing from sources with different impedances.

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1. Problem to be solved

The signal voltages that can be observed on a cable are not only dependent on the available power of the various sources, but also on the individual source and load impedances. The output voltage of a (100 Ω) ADSL disturber into a 150 Ω cable is about **1.58 dB** higher then when it is connected to a 100 Ω cable, while the power is about 0.18 dB lower. On the other hand when its disturbance couples into the wirepair under test, a 135 Ω receiver will observe a higher crosstalk voltage than a 100 Ω receiver.
The question arises how to deal with the fact that signal levels and crosstalk coupling functions are impedance dependent. Currently this impedance mismatch is more or less ignored in xDSL performance tests, which may cause an inconsistent interpretation on how to evaluate the crosstalk noise levels. This issue would have been simple when all xDSL systems were 135 Ω systems, but the fact that ADSL is a 100 Ω system has complicated matters significantly.

Now ETSI-TM6 has come into the proces of defining additional ADSL noise models that are fully dedicated to the FDD variant of ADSL, it becomes important to have this impedance issue fully understood. The evaluation of simple equivalent disturber sources as replacement for more complicated disturber mixtures, as developed within FSAN[1], requires this understanding.
The aim of this contribution is to have the problem identified, and to have it solved.

2 Problem analysis

2.1 The FSAN crosstalk sum, at the victim side

When disturbing an xDSL modem under test by a technology mixture of many disturbers connected to other wire pairs, then the well-known FSAN crosstalk sum can be used for calculating the total impairment. This sum is not a linear sum, but a weighed sum, although the crosstalk mechanism is a linear network. This weighing is required to account for the *probability* that a disturber occupies the worst-case wire pair, the next to worst-case wire pair, etc.
This FSAN sum is evaluated at the receiver input of the (victim) modem under test.

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[1] A group of Telco's working together on xDSL. The mothod was develloped at the Bern meeting and lateron published in [2,3,4,5].

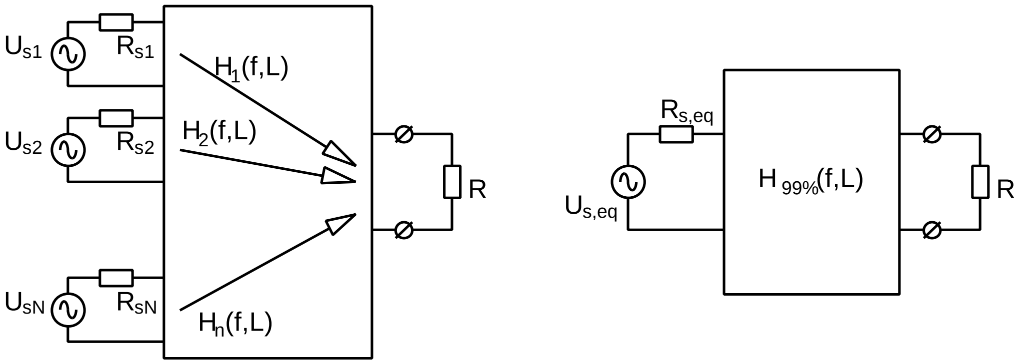
According to the superposition theorem of linear networks, it is not the (time dependent) power but the (time dependent) voltage that adds at the receiver input of the victim modem under test. In practice there is no need to predict the signal levels as a function of the time; some average value or 99% limit value is often adequate. Because all disturbers are assumed to be *uncorrelated*, the square of rms-values of the individual coupled voltages at the receiver input will add on a linear basis.
Since the impedance at that point is the same for all coupled voltages (it equals the input impedance of the receiver, which is 100Ω for ADSL), it is irrelevant if the crosstalk sum is presented as a rms *voltage* sum or a *power* sum. That's why the two different notations in table 1 (in terms of voltages or in terms of powers) have full equivalence. The FSAN crosstalk sum, in term of powers, was the notation used from the beginning, so this has become the common way of expressing this crosstalk sum.

FSAN crosstalk sum	
in terms of voltages:	$U^2_{x,tot} = (U_{x1}^{2\cdot Kn} + U_{x2}^{2\cdot Kn} + U_{x3}^{2\cdot Kn} + U_{x4}^{2\cdot Kn} + \ldots)^{1/Kn}$
in terms of powers:	$P_{x,tot} = (P_{x1}^{Kn} + P_{x2}^{Kn} + P_{x3}^{Kn} + P_{x4}^{Kn} + \ldots)^{1/Kn}$
	K_n=1/0.6 as default value, or a (slightly) different value if proven to be more appropriate for a particular cable

Table 1. $P_{x,n}$ (or $U_{x,n}$) is the power (or rms voltage) at the victims receiver input coupled from an individual disturber, when the crosstalk coupling function equals the 99% limit of all possible crosstalk coupling functions in that particular cable.
$P_{x,tot}$ (or $U_{x,tot}$) is the total power (or rms voltage) at the victims receiver input coupled from all disturbers, when they occupy ad random all possible crosstalk coupling functions in that particular cable.

2.2 The generalized crosstalk sum, at the disturber side

A more generalized usage of the original FSAN crosstalk sum has slipped in by summing the crosstalk at the *disturber* side in stead of at the *victims* receiver side. This generalized approach has been introduced without being noticed for the SDSL [6,7] noise models, and is also used for the VDSL [8] and ADSL [9] noise models. In this approach all disturbers are combined into two equivalent disturbers that are coupled to the victim modem by a single (99% limit) crosstalk coupling function (one for NEXT and one for FEXT).
In this approach all statistics of individual crosstalk coupling functions have been incorporated into the equivalent disturbers as well, so that the multi disturber model in figure 1a can be replaced by the equivalent disturber model in figure 1b because they produce the same impairment to the victim modem under test.



(a) multi disturber model *(b) equivalent disturber model*

Figure 1 Concept of an equivalent disturber.

The main advantage of using the equivalent disturber model over the multi disturber model is that it simplifies matters significantly without losing the possibility of modelling crosstalk coupling that is frequency, length and cable dependent. By using more than one equivalent disturber, one for each

location where several disturbers are co-located, the simplified model can also support diferences between NEXT and FEXT.
The main problem so far is that in the generalized crosstalk sum (at the disturber side) all sources may have different source impedances, while this is not the case for the well known FSAN crosstalk sum (at the victim side). The influence of these impedances has been ignored when evaluating their level for the ETSI noise models [7,9]

2.2.1 Two plausible candidates for evaluating the equivalent disturber

The question is what shall be the output level of these equivalent disturbers, when they are to represent a mixture of co-located disturbers in the most plausible way. Each disturber can have a different source impedance, and each mixture is terminated with a cable impedance that is different for each testloop.

To find an answer to this question, table 2 illustrates two possible notations of the FSAN crosstalk sum, but now applied to evaluate the equivalent disturber in stead of the total crosstalk sum. The consequence ot this is that:

- signal levels are taken at the *disturber* side and not at *victim* side of the crosstalk coupling;
- the generalized crosstalk sum in terms of voltages and powers are not equivalent anymore.

Both formulas have ignored the impact of the mismatch between source and cable impedance, so both formulas are approximations and both notations are candidates for the most plausible way to generalize the FSAN crosstalk sum.

FSAN crosstalk sum	***applied to the equivalent disturber while ignoring impedances***
in terms of powers:	$P_{s,eq} = (P_{s1}^{Kn} + P_{s2}^{Kn} + P_{s3}^{Kn} + P_{s4}^{Kn} + \ldots)^{1/Kn}$
in terms of voltages:	$U^2_{s,eq} = (U_{s1}^{2\cdot Kn} + U_{s2}^{2\cdot Kn} + U_{s3}^{2\cdot Kn} + U_{s4}^{2\cdot Kn} + \ldots)^{1/Kn}$
	K_n=1/0.6 as default value, or a (slightly) different value if proven to be more appropriated for a particular cable

Table 2. Two plausible candidates for crosstalk summing

2.2.2 How different are these two plausible candidates?

When all impedances are equal the two plausible candidates are identical, but this will not hold in a multi-impedance disturber environment. The internal rms voltage of the equivalent disturber equals $U_{eq} = 2 \cdot \sqrt{(P_{eq} \cdot R_{eq})}$, as explained in the next chapter. Then the FSAN crosstalk sum, generalised in terms of voltages, would results in an equivalent disturber having the following equivalent power:

$$U^2_{s,eq} = (U_{s1}^{2\cdot Kn} + U_{s2}^{2\cdot Kn} + U_{s3}^{2\cdot Kn} + U_{s4}^{2\cdot Kn} + \ldots)^{1/Kn}$$

$$P_{s,eq} \cdot R_{eq} = ((P_{s1} \cdot R_{s1})^{Kn} + (P_{s2} \cdot R_{s2})^{Kn} + (P_{s3} \cdot R_{s3})^{Kn} + (P_{s4} \cdot R_{s4})^{Kn} + \ldots.)^{1/Kn}$$

$$P_{s,eq} = ((P_{s1} \times R_{s1}/R_{eq})^{Kn} + (P_{s2} \times R_{s2}/R_{eq})^{Kn} + (P_{s3} \times R_{s3}/R_{eq})^{Kn} + (P_{s4} \times R_{s4}/R_{eq})^{Kn} + \ldots.)^{1/Kn}$$

In the case that a mixture of 135 Ω disturbers (ISDN, HDSL, SDLS) and 100 Ω disturbers (ADSL), is represented by one 135 Ω equivalent disturber, this formula will simplifies into:

$$P_{s,eq} = (P_{ISDN}^{Kn} + P_{HDSL}^{Kn} + (P_{ADSL} \times 100/135)^{Kn} + P_{SDSL}^{Kn} + \ldots)^{1/Kn}$$

FSAN crosstalk sum	***which one is right in a multi-impedance disturber environment?***
in terms of powers:	$P_{s,eq} = (P_{ISDN}^{Kn} + P_{HDSL}^{Kn} + P_{ADSL}^{Kn} + P_{SDSL}^{Kn} + \ldots)^{1/Kn}$
in terms of voltages:	$P_{s,eq} = (P_{ISDN}^{Kn} + P_{HDSL}^{Kn} + (P_{ADSL} \times 100/135)^{Kn} + P_{SDSL}^{Kn} + \ldots)^{1/Kn}$
	K_n=1/0.6 as default value, or a (slightly) different value if proven to be more appropriated for a particular cable

Table 3. Difference between power and voltage summing

In the general case, voltages and not powers do add in linear networks. In our case, when the rms average of uncorrelated sources is to be evaluated, this superposition simplifies into adding the square of the rms voltages. For this reason the voltage method has been used in [7,9] for calculating the levels

of the equivalent disturbers, without making any further notice[2] on the method and the impedance mismatch.
When all sources have equal source impedance, the generalization of the FSAN crosstalk sum in terms of voltages or powers yield the same result; in that case is the impedance issue hardly any issue. In a multi-impedance disturber environment however, like ADSL (100Ω) and SDSL, HDSL, ISDN (all 135Ω) there is a difference that can be significant. The voltage approach causes that ADSL appears to be a more silent disturber compared to the power approach. The difference is a factor $\sqrt{(100/135)}$ in voltage, which is about 1.3 dB.

Now it has been clear how much difference the two evaluation methods produce, it becomes clear what problem has to be solved in this contribution.

3 Problem solution, under mismatched conditions

This section shows which of the two plausible candidates for crosstalk summing at the disturber side is the correct one. To enable this, we have to start from the basic definitions regarding mismatched sources.

3.1 Individual noise levels at the output of xDSL disturbers: definitions

From a crosstalk point of view, a disturber produces noise. Figure 2 illustrates the Thevenin equivalent of such an xDSL disturber source, having output impedance of R_s, that is terminated with load impedance R (representing the loaded cable impedance). For reasons of simplicity these impedances are assumed to be real, but the rational behind this discussion can be generalized to frequency dependent complex impedances.

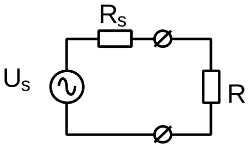


Figure 2 Thevenin equivalent of disturber.

We use the following definitions:

- Let U be the rms output voltage over load R
- Let P be the output power absorbed by load R
- Let U_s be the available rms output voltage that can be consumed from this source, which is the voltage over load impedance R=∞. This equals the internal voltage of the source.
- Let P_s be the available output power that can be consumed from this source, which is the power into load impedance $R=R_s$

When P_s and R_s are given quantities (which is the common way how xDSL signal levels are defined), the following relations hold:

$P_s = ¼·U_s^2/R_s$	$P = U^2/R$	$P = (4·R·R_s)/(R+R_s)^2 \times P_s$	$P = (R)/(R+R_s)^2 \times U_s^2$
$U_s = 2 ·\sqrt{(P_s·R_s)}$	$U =\sqrt{(P·R)}$	$U = (2·R)/(R+R_s) \times \sqrt{(P_s· R_s)}$	$U = (R)/(R+R_s) \times U_s$

To illustrate the numerical impact of above formulas, the table below shows several examples of how much the output voltage and power of an ADSL modem changes when it is terminated with an impedance that differs from 100 Ω.

R_s=100 W	D**P** **$P = (4·R·R_s)/(R+R_s)^2 \times P_s$**	D**U** **$U = (R)/(R+R_s) \times U_s$**
R =100 Ω	ΔP = 0 dB	ΔU = 0 dB
R =110 Ω	ΔP = -0.0099 dB	ΔU = +0.4041 dB
R =120 Ω	ΔP = -0.0360 dB	ΔU = +0.7558 dB
R =135 Ω	ΔP = -0.0974 dB	ΔU = +1.2059 dB
R =150 Ω	ΔP = -0.1773 dB	ΔU = +1.5836 dB

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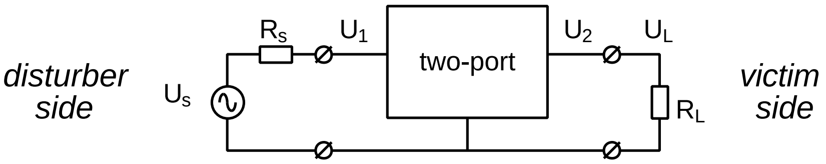
[2] The original text suggest that the FSAN crosstalk sum was evaluated in terms of powers, but this is not what has been implemented in the software that was used for the calculations.

3.2 Individual noise levels at the input of victim xDSL modem

Each disturber impairs the victim xDSL modem under test by coupling noise into its receiver input. From a single disturber point of view the following equivalent circuit diagram applies for this coupling:



Annex A studies the transfer between disturber and victim, in the case of impedance mismatch and an arbitrary linear two-port (that is reciprocal and symmetrical). It compares various approximations for this transfer function, by evaluating the error of each approximation due to impedance mismatch. The result of the study in annex A is an approximation formula that is simple and still reasonable accurate for the majority of European cables. These results can be summarized as follows:

- Let U_s and P_s be the available rmsvoltage and power of the disturber, having source impedance R_s.
- Let U_L and P_L be the coupled rms voltage and power to the victim, having a load impedance R_L.
- Let S_x be the transfer function that is measured when source and load impedances are equal to a (chosen) reference impedance R_n. This quantity equals the transmission s-parameter, normalized to R_n, . The value $1/S_x$ equals the insertion loss between two impedances R_n.

Mark that this R_n is in no way related to the characteristic impedance of the cable under test, nor the source and load impedances (R_s and R_L) of various xDSL modems. A value of R_n=135Ω is used within ETSI since it represents a fair average among the wide range of cables that are most commonly used in Europe (mainly between 100-150Ω).

The study in annex A concludes that the following approximation of crosstalk coupling remains fair under mismatched conditions within reasonable accuracy (within 0.3 dB for cables ranging from 100..150Ω, characterized at 135Ω) and reasonable simplicity.

$\boxed{\frac{P_L}{P_s} \approx \sqrt{S_x}}$ which is fully equivalent to $\boxed{\frac{U_L}{½ \cdot U_s} \approx S_x \times \sqrt{R_L / R_s}}$

From now on we will use this approximation for generalizing the FSAN crosstalk sum.

3.3 Generalized crosstalk sum, in a multi-impedance environment

By shifting the crosstalk summation from the *victim* side of the crosstalk coupling network to the *disturber* side, we generalize the crosstalk sum. The result is an equivalent disturber, that represents all colocated disturbers as if it was a single disturber on its own.
Since all disturbers are assumed to be uncorrelated, the square of all rms voltages at the victim side will add. Because all individual contributions are terminated into the same victim impedance, this addition is equivalent to adding powers at the victim side.

By using the prefered approximation of crosstalk coupling for a single disturber under mismatched conditions, the crosstalk sum at the *victim* side can be expressed in its individual contributions. If more disturbers are involved, the superposition theorem can be used at the victim side to express the total impairment as a summation of the individual contributions.
For a particular wirepair under test (labeled with "r"), and a particular distribution of disturbers over the other wirepairs (labeled with "k"), the crosstalk sum at the victim side can now be approximated (within 0.3 dB for European cables) by two fully identical expressions:

$$
\begin{aligned}
P_{L,\{r\}} &\approx \Sigma_k\, S_{x,\{k,r\}}^2 \cdot P_{s,\{k\}} \\
U_{L,\{r\}}^2 &\approx \Sigma_k\, S_{x,\{k,r\}}^2 \cdot (R_L/R_{s,\{k\}}) \cdot U_{s,\{r\}}^2
\end{aligned}
$$

In this formula represents $S_{x,\{k,r\}}$ the absolute value of the actual crosstalk coupling from wire pair "k" to wire pair "r", measured against R_n=135Ω for a particular frequency.

Since xDSL modems are connected to wire pairs in arbitrary combinations, the crosstalk sum that will not be exceeded in 99% of the statistical cases is more relevant than its average. This value is named the 99% limit of expected values, and is denoted here as $P_{L,99\%}$ or $U_{L,99\%}$ or $\text{á...ñ}_{99\%}$. Mark that this "99%" value is not equal to the "expectation" value in the classical statistical sense, allthough the notation is look alike.

$$
\begin{aligned}
P_{L,99\%} &= \text{á}P_{L,\{r\}}\text{ñ}_{99\%} &&= \text{á}\Sigma_k\, S_{x,\{k,r\}}{}^2 \cdot P_{s,\{k\}}\text{ñ}_{99\%} \\
U_{L,99\%}{}^2 &= \text{á}U_{L,\{r\}}{}^2\text{ñ}_{99\%} &&= \text{á}\Sigma_k\, S_{x,\{k,r\}}{}^2 \cdot (R_L/R_{s,\{k\}}) \cdot U_{L,\{r\}}{}^2\text{ñ}_{99\%}
\end{aligned}
$$

Both expressions are identical, and can both be simplified into a crosstalk summation formula that requires only the *number* of similar disturbers. The calculation of such a simplification is an extensive statistical exercise, which is beyond the scope of this contribution, but we can reuse results from the past dedicated to simpler conditions.

Homogeneous disturber case

The homogeneous disturber case, where all disturbers are equal (in signal level and impedance), has been investigated by Unger [11] around 1985 and later on within the ANSI T1E1.4 working group[3] around 1994-1995. This resulted in the statistical observation that when N systems are ad random connected to wire pairs, the 99% crosstalk powersum is proportionally with $N^{0.6}$. Both the ANSI ADSL standard [10] (Annex B1) and Unger [11] are reporting an empirical value of $N^{0.6}$.
Similar experiments within KPN on a few KPN cables have resulted in different[4] observations ranging between $N^{0.65}$ and $N^{0.8}$, over a wide range of N. This illustrates that this empirical formula is cable dependent, and that a generalized formulation $N^{1/Kn}$, as introduced in [7], is more appropriate. The value K_n=1/0.6 serves only as default value.
In formula, this observation can be expressed as:

$$\frac{\left\langle \sum_{k=1}^{N} S_{x,\{k,r\}}{}^2 \times P_s \right\rangle_{99\%}}{N^{1/K_n}} \approx \begin{Bmatrix} independent \\ from \cdot N \end{Bmatrix} \quad (K_n\text{=empirical value; on default } K_n\text{=1/0.6})$$

By using this observation the so called "99% crosstalk curve" has been defined as:

$$S_{x,99\%} = \frac{1}{N^{1/K_n}} \times \left\langle \sum_{k \in \{N\}} S_{x,\{k,r\}}{}^2 \right\rangle_{99\%} \quad (K_n\text{=empirical value; on default } K_n\text{=1/0.6})$$

In this formula the constant exponent is replaced by a factor, to expand the applicability of this formula. It enables us to adapt this factor when the 99% limit is replaced by another limit (e.g. 90%) or to fine tune this factor to a particular cable of interest. The default value for this factor equals K_n=1/0.6.

The expression for the crosstalk sum, observed at the victim side has now been simplified to:

$P_{L,99\%} \approx (S_{x,99\%})^2 \times P_s \times N^{1/Kn}$ *when all disturbers are equal*

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[3] The empirical formula $N^{0.6}$ has popped-up in one of the ANSI-ADSL drafts that resulted in the ADSL standard [10], and since then it became commonly used within FSAN and ETSI. It is assumed that this was based on the observations made by Unger in [11].

[4] Unger reports in [11] a numerical experiment, using statistical assumptions fitted to measurements on four cables. The standard deviation of the individual crosstalk coupling functions within these four cables was observed as 11.77 dB (based on previous work of S.H. Lin). Different cables may have different standard deviations in crosstalk coupling and therefore different dependencies between power sum and number of disturbers.

By defining the equivalent disturber as a source with an impedance equal to the chosen reference impedance R_n, we get for the homogeneous crosstalk sum at the source and at the load side:

$P_{s,eq} = P_s \times N^{1/Kn}$	$U_{s,eq}^{2} = (U_s^2 \times R_n/R_s) \times N^{1/Kn}$
$P_{L,99\%} \approx (S_{x,99\%})^2 \times P_{s,eq}$	$U_{L,99\%} \approx S_{x,99\%} \times U_{s,eq}$

when all disturbers are equal

Mixed disturber case

The FSAN crosstalk sum was developed to handle the inhomogeneous disturber case, at the victim side of the crosstalk network. The contribution of individual disturbers to the total impairment at the victim side is evaluated by assuming that the crosstalk coupling of each disturber follows the 99% crosstalk curve. Since only one disturber can be connected to that virtual "poor case" wire pair, the total impairment is a weighed sum of the individual contributions at the victim side. This weighed sum equals:

$$P_{L,99\%} \approx [\, (P_{L1,99\%})^{Kn} + (P_{L2,99\%})^{Kn} + ... + ... + <\text{all other disturbers}>]^{1/Kn}$$

$$P_{L,99\%} \approx [\, N_{isdn} \times (P_{L,isdn,99\%})^{Kn} + N_{adsl} \times (P_{L,adsl,99\%})^{Kn} + ...]^{1/Kn}$$

$$P_{L,99\%} \approx [\, (P_{L,isdn,99\%} \times N_{isdn}^{1/Kn})^{Kn} + (P_{L,adsl,99\%} \times N_{adsl}^{1/Kn})^{Kn} + ...]^{1/Kn}$$

for a single class of N disturbers (e.g. ADSL), all having the same impedance, holds:

$P_{L,isdn,99\%} \approx (S_{x,99\%})^2 \times P_{s,isdn}$ (e.g. 135 Ω source impedance)
$P_{L,adsl,99\%} \approx (S_{x,99\%})^2 \times P_{s,adsl}$ (e.g. 100 Ω source impedance)

Since the power coupling, measured at the reference impedance (e.g. 135Ω) is observed as the best approximation for the transfer, measured at arbitary source and load impedance, the crosstalk sum at the victim side can now be expressed as:

$$P_{L,99\%} \approx [\, ((S_{x,99\%})^2 \times P_{s,isdn} \times N_{isdn}^{1/Kn})^{Kn} + ((S_{x,99\%})^2 \times P_{s,adsl} \times N_{adsl}^{1/Kn})^{Kn} + ...]^{1/Kn}$$

$$P_{L,99\%} \approx (S_{x,99\%})^2 \times [\, (P_{s,isdn} \times N_{isdn}^{1/Kn})^{Kn} + (P_{s,adsl} \times N_{adsl}^{1/Kn})^{Kn} + ...]^{1/Kn}$$

$$P_{L,99\%} \approx (S_{x,99\%})^2 \times [\, (P_{s,isdn})^{Kn} \times N_{isdn} + (P_{s,adsl})^{Kn} \times N_{adsl} + ...]^{1/Kn}$$

since $P_{L,99\%} = (S_{x,99\%})^2 \times P_{s,eq}$ because all impedances are equal, we get:

$$P_{s,eq} \approx [\, (P_{s,isdn})^{Kn} \times N_{isdn} + (P_{s,adsl})^{Kn} \times N_{adsl} + ...]^{1/Kn}$$

By defining the expression between brackets [..] as the available power $P_{s,eq}$ of the equivalent disturber, we can create a convenient way to describe the crosstalk sum for the mixed disturber case. By definition, this source has an impedance equal to the chosen reference impedance R_n, which value was also used to evaluate the crosstalk transfer functions S_x.

available power of equivalent disturber: $P_{s,eq} = [\,(P_{s,isdn})^{Kn} \times N_{isdn} + (P_{s,adsl})^{Kn} \times N_{adsl} + \ldots\,]^{1/Kn}$ $= [\,(P_{s,isdn} \times N_{isdn}^{1/Kn})^{Kn} + (P_{s,adsl} \times N_{adsl}^{1/Kn})^{Kn} + \ldots\,]^{1/Kn}$ $= [\,(P_{s,eq,isdn})^{Kn} + (P_{s,eq,adsl})^{Kn} + (P_{s,eq,sdsl})^{Kn} + \ldots\,]^{1/Kn}$
crosstalk power, at victim side of modem under test $P_{L,99\%} \approx (S_{x,99\%})^2 \times P_{s,eq}$
source voltage of equivalent disturber, having source impedance R_n $U_{s,eq}^2 = [\,(U_{s,isdn}^2 \times R_n/R_{s,isdn})^{Kn} \times N_{isdn} + (U_{s,adsl}^2 \times R_n/R_{s,adsl})^{Kn} \times N_{adsl} + \ldots\,]^{1/Kn}$ $= [\,(U_{s,isdn}^2 \times R_n/R_{s,isdn} \times N_{isdn}^{1/Kn})^{Kn} + (U_{s,adsl}^2 \times R_n/R_{s,adsl} \times N_{adsl}^{1/Kn})^{Kn} + \ldots\,]^{1/Kn}$ $= [\,(U_{s,eq,isdn}^2)^{Kn} + (U_{s,eq,adsl}^2)^{Kn} + (U_{s,eq,sdsl}^2)^{Kn} + \ldots\,]^{1/Kn}$
crosstalk volage, at victim side of modem under test $U_L \approx S_{x,99\%} \times U_{s,eq} \times \sqrt{(R_L/R_n)}$
R_n= chosen reference impedance = 135W *R_s = output impedance of the disturber (ADSL=100 W; SDSL=135W)* *R_L = input impedance of the victim receiver under test.* *$S_{x,99\%}$ = 99% limit of crosstalk coupling functions* *K_n=1/0.6 as default value, or a (slightly) different value if proven to be more appropriated for a particular cable or other value than 99% limit*

Table 4. Generalized crosstalk sum in a multidimensionale environement

The fortunate choice in the past by accidentaly describing the FSAN crosstalk sum in terms of powers to evaluate equivalent disturbers [7,8,9] without bothering any impedance mismatch, may give a feel of recognition. The look and feel of the *generalized* crosstalk sum at disturber and victim side of the crosstalk coupling network are the same. The difference is that it has now been well defined which power has to be used, and that the evaluation of this sum has been shifted from victim side to disturber side of the crosstalk coupling.

4 Usage of the generalized crosstalk sum

This section illustrates how to use the generalized crosstalk sum in practice by discussing two applications. The first one is related to the evaluation of the ADSL noise models, and the second one on how to evaluate the total impairment at the terminals of the ADSL modem under test, when calculating the performance of that ADSL modem.

4.1 Equivalent disturbers for ADSL performance tests

This example starts from the rationale behind the ADSL noise models, as have been described in [9] and supported by FSAN. In that contribution, the technology mix was defined directly in terms of equivalent disturbance per technology, but each one at its own impedance. We start here from that point as well. The individual power levels of noise model C are referenced as:

P_1 = equivalent disturbance of 135Ω ISDN.2B1Q systems, occupying 10 wirepairs
P_2 = equivalent disturbance of 135Ω HDSL.2B1Q systems, occupying 4 wirepairs
P_3 = equivalent disturbance of 100Ω ADSL systems, occupying 15 wirepairs
P_4 = equivalent disturbance of 135Ω SDSL systems, occupying 15 wirepairs
P_5 = equivalent disturbance of 130Ω ISDN-PRI/HDB3 systems, occupying 4 wirepairs

The associated internal voltages, when each equivalent disturber is expressed as source with its originating impedance or transformed into an equivalent source normalized to 135Ω are:

$U_{1\,(at\ 135\Omega)} = 2\times\sqrt{(P_1\times135)}$ ==> $U_{eq1\,(at\ 135\Omega)} = 2\times\sqrt{(P_1\times135)} = U_1\times\sqrt{(135/135)}$
$U_{2\,(at\ 135\Omega)} = 2\times\sqrt{(P_2\times135)}$ ==> $U_{eq2\,(at\ 135\Omega)} = 2\times\sqrt{(P_2\times135)} = U_2\times\sqrt{(135/135)}$
$U_{3\,(at\ 100\Omega)} = 2\times\sqrt{(P_3\times100)}$ ==> $U_{eq3\,(at\ 135\Omega)} = 2\times\sqrt{(P_3\times135)} = U_3\times\sqrt{(135/100)}$
$U_{4\,(at\ 135\Omega)} = 2\times\sqrt{(P_4\times135)}$ ==> $U_{eq4\,(at\ 135\Omega)} = 2\times\sqrt{(P_4\times135)} = U_4\times\sqrt{(135/135)}$
$U_{5\,(at\ 130\Omega)} = 2\times\sqrt{(P_5\times130)}$ ==> $U_{eq5\,(at\ 135\Omega)} = 2\times\sqrt{(P_5\times135)} = U_5\times\sqrt{(135/130)}$

In [9], the source impedance R_{eq} of this combined equivalent disturber was choosen equal to the reference impedance $R_{eq}=R_n=135\Omega$, which is seen as a fair average of commonly used cables in Europe. When P_{eq} is the available power of this source (available when terminated with R_{eq}), en when U_{eq} is the internal voltage of this source, then their most plausible values is summarized below. All expressions are equivalent, so it is a matter of taste which one is the most convenient one.

$$
\begin{aligned}
P_{eq} &= \left(P_1^{K_n}+P_2^{K_n}+P_3^{K_n}+P_4^{K_n}+P_5^{K_n}\right)^{1/K_n} && = ¼\cdot U_{eq}^2/135 \quad (eq1)\\
U_{eq}^2 &= \left(U_{eq1}^{2\cdot K_n}+U_{eq2}^{2\cdot K_n}+U_{eq3}^{2\cdot K_n}+U_{eq4}^{2\cdot K_n}+U_{eq5}^{2\cdot K_n}\right)^{1/K_n} && = 4\times P_{eq}\times 135 \quad (eq2) \qquad K_n=1/0.6\\
U_{eq}^2 &= \left(U_1^{2\cdot K_n}+U_2^{2\cdot K_n}+\left(U_3^2\times\tfrac{135}{100}\right)^{K_n}+U_4^{2\cdot K_n}+\left(U_5^2\times\tfrac{135}{130}\right)^{K_n}\right)^{1/K_n} && = 4\times P_{eq}\times 135 \quad (eq3)
\end{aligned}
$$

Table 5. Three identical expressions to evaluate the equivalent disturber level

4.2 Impairment level of ADSL performance tests

Figure 3 illustrates a functional diagram of an impairment generator, as it is commonly used for ADSL, SDSL and VDSL testing.

- It is based on two equivalent disturbers; one for all disturbers co-located at the NT side of the testloop and one for all disturbers co-located at the LT side. Eaxh equivalent disturber has a value U_{eq} as internal voltage, P_{eq} as available power and R_{eq} as (virtual) source impedance.
- The transfer functions $H_1(f,L)$ and $H_2(f,L)$ represent the 99% NEXT and FEXT coupling functions, that are dependent on the frequency *f* and length L. When measuring this crosstalk mechanism as two-port, by terminating all unused cable ends by 135Ω, then each transfer functions H(f,L) equals the s_{21} parameter of this two-port, normalized to a reference impedance of 135Ω.
- The impairment generator injects noise into the victim modem under test, which is a 100Ω ADSL modem in this example. The noise level that can be observed at the terminals of that modem under test is refered to as voltage U_x over resistance R_x or power P_x into resistance R_x.

The signal level of this injected noise will be:

$$
\begin{aligned}
P_x &= H_1(f,L)^2\times P_{eq1}+H_2(f,L)^2\times P_{eq2} && = U_x^2/R_x\\
U_x^2 &= \left(\left(H_1(f,L)\times U_{eq1}\right)^2+\left(H_2(f,L)\times U_{eq2}\right)^2\right)\times\left(\tfrac{R_x}{R_n}\right) && = P_x\times R_x
\end{aligned}
$$

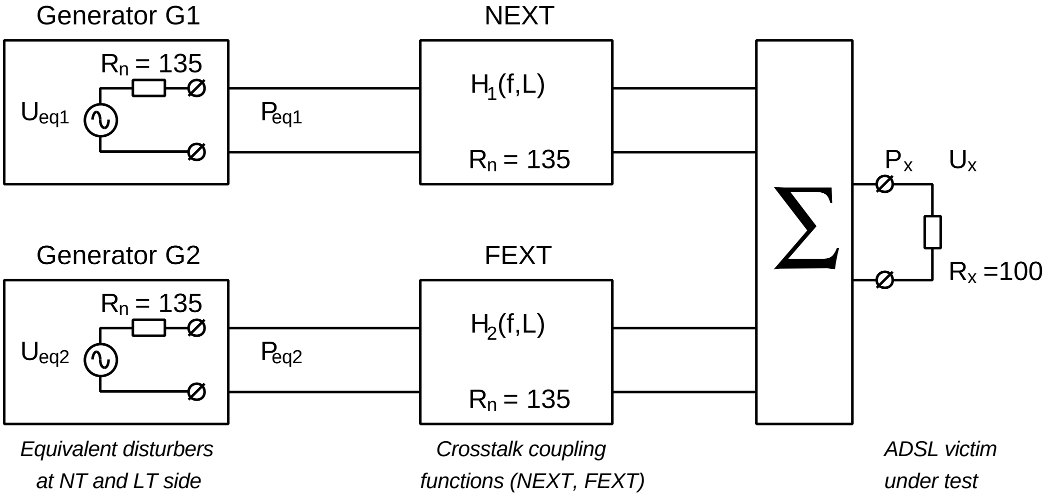


Figure 3. Functional diagram of combing a NEXT and FEXT equivalent disturber into a compound crosstalk level.

5 Conclusions

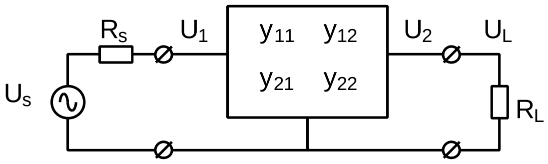
For crosstalk summing at the *victim* side of a modem under test, both the power method and the voltage method provide equal results since all signal is added at the same impedance. The well known FSAN crosstalksum was formulated for this special case, and is still considered as adequate.
For crosstalk summing at the *disturber* side of a modem under test, to combine many individual disturbers into a single equivalent disturber, the power and voltage methods produce different results when disturbers with different impedances are combined. Difference between the two methods of more then 1.3 dB are caused by this effect when evaluating the equivalent disturbers for the ADSL performance test.

The introduction of using equivalent disturbers for the noise models of ETSI xDSL standards has simplified the noise models significantly but the impact of multiple impedances was ignored so far. This caused different interpretations on how to use the FSAN crosstalk sum for this multi-impedance case, resulting in a power and voltage summing method.
Both the power and the voltage summing method are approximations only, which ininherent to their simplicity by ignoring the cable impedance.

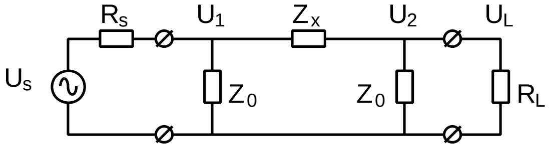
This contribution has identified this impedance discrepancy, and proposes a more generalized crosstalk summing. It has been demonstrated here that the consistent usage of the *power* method gives plausible results (error <0.3 dB) for cables ranging between 100-150 Ω, characterized at 135 Ω and connected to modems ranging from 100-135 Ω. The advantage is that cable impedance is ignored completely with this approximation.
==> *The power method is a good compromise between simplicity and accuracy within a limit range*
Under more significant mismatch conditions, the power method becomes too inaccurate, and other approximations are required that also account for the cable impedance.

Appendix A: Approximated transfer

Assume, a disturber (U_s, R_s) that is weakly coupled to a resistive load and that induces a voltage (U_L) over load impedance (R_L). This crosstalk coupling is a linear process, so this process can alway be modelled in terms of two-port matrix parameters such a s-parameters, ABCD parameters or y-parameters, as illustrated below.



Each linear two-port can be described by numerous equivalent circuit diagrams. Since this two-port is reciprocal and in general symmetrical in nature, this diagram simplifies significantly. In that case, $y_{12} = y_{21}$ due to reciprocity, and $y_{11} = y_{22}$ due to symmetry, and an equivalent Π-diagram will simplify into the diagram shown below:



$Z_x = –1/y_{12}= –1/y_{21}$

$Z_0 = 1/(y_{11}+y_{12}) = 1/(y_{22}+y_{21})$
0

In this Π-model represents Z_0 a virtual impedance that is very close to the characteristic impedance op the cable (100~150 ohm), and Z_x a virtual impedance that is significantly higher in magnitude then Z_0. (when $| Z_x| = 50·| Z_0|$, then the crosstalk coupling is about –40 dB)

It can be demonstrated, by some computational effort, that the transfer function H from U_s to U_L equals:

$$H = \frac{U_L}{½ \cdot U_s} = \frac{2}{\left(R_s/Z_0 + 1\right)\cdot\left(Z_x/R_L + Z_x/Z_0 + 2\right) + \left(R_s/R_L - 1\right)}$$

The crosstalk coupling, measured at normalized source and load impedance R_n equals to this H by replacing all R_s and R_L by R_n.

$$\Longrightarrow S_x = \frac{2}{\left(R_n/Z_0 + 1\right)\cdot\left(Z_x/R_n + Z_x/Z_0 + 2\right)} = \left(\frac{R_n}{2 \cdot Z_x}\right) \times \frac{2}{\left(R_n/Z_0 + 1\right)\cdot\left((1 + R_n/Z_0)/2 + R_n/Z_x\right)}$$

This S_x is commonly normalized to 135Ω (within ETSI) since 135Ω is considered as the characteristic impedance of the "average" European cable.
Since the characteristic impedance of the majority of European cables ranges between 100 and 150Ω, with exceptional cases even up to 180Ω, it is prefered to have a simple calculation of the crosstalk coupling, that is a fair approximate for the whole range of relevant xDSL and cable impedances.

A.1 The analyzed approximations

A.1.1 Approximation "1", based on S_x (at R_n) only

A possible approximation of the transfer function *H* is based on the following rationale:

- Voltage $U_L \approx (½ \cdot U_s) \cdot S_x$
- ==> $H \approx S_x$

This approximation ignores all mismatch corrections, and assumes that the reference impedance R_n is choosen to be close to Z_0, R_s and R_L. The only information that is required is S_x (at unknown reference impedance)

A.1.2 Approximation "2", based on S_x (at R_n), R_s and R_L

Another possible approximation of the transfer function *H* is based on the following rationale:

- Power $P_s \approx ¼ \cdot U_s^2/R_s$
- Power $P_L \approx U_L^2/R_L$
- Transfer $P_L \approx S_x^2 \cdot P_s$
- ==> $H \approx S_x \cdot \sqrt{(R_L/R_s)}$

This approximation ignores all mismatch corrections, and assumes that the reference impedance R_n is choosen to be close to Z_0, R_s and R_L. The only information that is required is S_x (at unknown reference impedance) and the source and load impedances.

A.1.3 Approximation "3", based on S_x (at R_n), R_s and R_L

A third approximation of the transfer function *H* is based on the following rationale:

- The the cable impedance equals the choosen reference impedance R_n
- Voltage $U_1 \approx (½ \cdot U_s) \cdot (2 \cdot R_n)/(R_n+R_s)$
- Voltage $U_2 \approx U_1 \cdot S_x \cdot (2 \cdot R_L)/(R_L+R_n)$
- ==> $H \approx S_x \cdot (2 \cdot R_n)/(R_n+R_s) \cdot (2 \cdot R_L)/(R_L+R_n)$

This approximation tries to account for the mismatch at input and output, assuming that the cable impedance is R_n.
The only information that is required is S_x (at known reference impedance) and the source and load impedances.

A.1.4 Approximation "4", based on S_x (at R_n), R_S, R_L and Z_0

A fourth approximation of the transfer function *H* is based on the following rationale:

- The input impedance of the cable equals its characteristic impedance ($\approx Z_0$)
- Voltage $U_1 \approx (½ \cdot U_s) \cdot (2 \cdot Z_0)/(Z_0+R_s)$
- Voltage $U_2 \approx U_1 \cdot S_x \cdot (2 \cdot R_L)/(R_L+Z_0)$
- ==> $H \approx S_x \cdot (2 \cdot Z_0)/(Z_0+R_s) \cdot (2 \cdot R_L)/(R_L+Z_0)$

This approximation tries to account for the mismatch at input and output, assuming that the cable impedance is Z_0.
The only information that is required is S_x (at unknown reference impedance) and the source, load and cable impedances.

A.1.5 Other approximations

There are many more possibilities to approximate *H*. The alternatives that have been considered during this study were more complicated the above ones, while numerical comparison with the true transfer *H* did not demonstrate any advantage over the above ones.

A.2 Comparing the accuracy of the approximations.

Table 6 and 7 shows the difference between the true transfer function and the approximated transfer function for each of the 4 four approximation formulas at varioius source and load impedances. This example is evaluated at about -40 dB crosstalk coupling ($Z_x = 50 \times Z_0$) and all real impedances. The reference impedance was chosen to be R_n=135 Ω.
The approximated values that are closer than 0.35 dB to the true value are highlighted in gray. Table 7 is dedicated to extreme mismatch conditions, and has chosen a return loss of RL=3dB as the worst case mismatch.

[1]	**[2]**	**[3]**	**[4]**	***75 W cable***
-0.585	-0.585	-0.780	-0.764	**R_s=100, R_L=100**
1.011	-0.292	-0.390	-0.752	**R_s=135, R_L=100**
-1.596	-0.292	-0.390	-0.752	**R_s=100, R_L=135**
0.000	0.000	0.000	-0.740	**R_s=135, R_L=135**
				100 W cable
-0.220	-0.220	-0.415	-0.220	**R_s=100, R_L=100**
1.193	-0.110	-0.207	-0.207	**R_s=135, R_L=100**
-1.413	-0.110	-0.207	-0.207	**R_s=100, R_L=135**
0.000	0.000	0.000	-0.195	**R_s=135, R_L=135**
				135 W cable
0.169	0.169	-0.025	-0.025	**R_s=100, R_L=100**
1.388	0.085	-0.013	-0.013	**R_s=135, R_L=100**
-1.219	0.085	-0.013	-0.013	**R_s=100, R_L=135**
0.000	0.000	0.000	0.000	**R_s=135, R_L=135**
				150 W cable
0.305	0.305	0.110	-0.049	**R_s=100, R_L=100**
1.456	0.153	0.055	-0.037	**R_s=135, R_L=100**
-1.151	0.153	0.055	-0.037	**R_s=100, R_L=135**
0.000	0.000	0.000	-0.024	**R_s=135, R_L=135**
				180 W cable
0.536	0.536	0.341	-0.204	**R_s=100, R_L=100**
1.571	0.268	0.171	-0.191	**R_s=135, R_L=100**
-1.035	0.268	0.171	-0.191	**R_s=100, R_L=135**
0.000	0.000	0.000	-0.179	**R_s=135, R_L=135**
				600 W cable
1.745	1.745	1.550	-4.455	**R_s=100, R_L=100**
2.176	0.873	0.775	-4.448	**R_s=135, R_L=100**
-0.431	0.873	0.775	-4.448	**R_s=100, R_L=135**
0.000	0.000	0.000	-4.441	**R_s=135, R_L=135**
				800 W cable
1.932	1.932	1.738	-6.134	**R_s=100, R_L=100**
2.270	0.966	0.869	-6.129	**R_s=135, R_L=100**
-0.337	0.966	0.869	-6.129	**R_s=100, R_L=135**
0.000	0.000	0.000	-6.123	**R_s=135, R_L=135**

Table 6 Accuracy of various approximations under normal mismatch conditions, when the cable is chartacterized at 135 Ω. The example with 600 and 800 Ω cables are indicative for the transfer at low frequencies (telephony band)

EXTREME MISMATCH

[1]	[2]	[3]	[4]	*100 W cable L>H*
-9.517	5.817	-0.318	-0.220	**R_s=17.1, R_L=584, (RL=–3dB)**
-5.809	1.181	-0.255	-0.212	**R_s=50, R_L=250**
-3.282	0.698	0.194	-0.184	**R_s=100, R_L=250**
-1.868	0.808	0.402	-0.171	**R_s=135, R_L=250**
-1.327	0.892	0.474	-0.167	**R_s=150, R_L=250**
0.268	1.237	0.665	-0.156	**R_s=200, R_L=250**
1.615	1.615	0.803	-0.147	**R_s=250, R_L=250**
5.932	5.932	1.639	-0.100	**R_s=17.1, R_L=584, (RL=–3dB)**
				100 W cable H>L
5.700	5.700	-2.278	-0.342	**R_s=17.1, R_L=17.1, (RL=–3dB)**
0.746	0.746	-1.314	-0.277	**R_s=50, R_L=50**
3.273	0.263	-0.864	-0.249	**R_s=100, R_L=50**
4.687	0.373	-0.657	-0.236	**R_s=135, R_L=50**
5.228	0.457	-0.585	-0.232	**R_s=150, R_L=50**
6.823	0.803	-0.394	-0.220	**R_s=200, R_L=50**
8.171	1.181	-0.255	-0.212	**R_s=250, R_L=50**
21.151	5.817	-0.318	-0.220	**R_s=584, R_L=17.1, (RL=–3dB)**
				150 W cable L>H
-9.311	6.019	-0.027	-0.015	**R_s=25.7, R_L=877, (RL=–3dB)**
-5.496	1.493	0.058	-0.036	**R_s=50, R_L=250**
-3.533	0.447	-0.057	-0.011	**R_s=100, R_L=250**
-2.382	0.294	-0.112	0.002	**R_s=135, R_L=250**
-1.932	0.286	-0.131	0.006	**R_s=150, R_L=250**
-0.581	0.388	-0.184	0.018	**R_s=200, R_L=250**
0.588	0.588	-0.224	0.027	**R_s=250, R_L=250**
6.144	6.144	-0.555	0.105	**R_s=877, R_L=877, (RL=–3dB)**
				150 W cable H>L
5.893	5.893	0.499	-0.136	**R_s=25.7, R_L=25.7, (RL=–3dB)**
2.398	2.398	0.338	-0.101	**R_s=50, R_L=50**
4.362	1.352	0.225	-0.075	**R_s=100, R_L=50**
5.513	1.199	0.169	-0.062	**R_s=135, R_L=50**
5.963	1.192	0.150	-0.058	**R_s=150, R_L=50**
7.314	1.293	0.097	-0.046	**R_s=200, R_L=50**
8.483	1.493	0.058	-0.036	**R_s=250, R_L=50**
21.350	6.019	-0.027	-0.015	**R_s=877, R_L=25.7, (RL=–3dB)**

Table 7 Accuracy of various approximations under extreme mismatch conditions, when the cable is characterized at 135 Ω.

Observations

- None of the approximations give exact results
- Method "4" is the best of therse 4 methods when Rn is chosen between 0.7×$|Z_0|$ ... 1.4×$|Z_0|$. The error is kept below 0.35 dB for real termination impedances, even at extreme mismatch conditions. This approach requires knowledge about Z_0
- Method "2" and "3" are not as good as method "4" and are similar in accuracy. Their advantage is that no information about Z_0 is required. Method "2" ,however is a slightly simpler then "3" and even a slightly better then "3" In the range of 100-150 ohm cables and 100-135 ohm systems.
- Method "1" is the simplest method of all, but has the lowest accuracy

Conclusion

The power method "2" is a good compromise between simplicity and accuracy. The accuracy is often better then 0.3 dB in the range of 100-150 ohm cables and for 100-135 ohm source and load impedances, provided that the cable transfer is normalized to a 135 Ω reference impedance. This is the application range for ETSI testloops and xDSL systems.

Appendix B: References

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