TITLE **Specification of crest distribution mask for noise in performance tests**

PROJECT VDSL, part 1 (Also HDSL and ADSL)

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STATUS For information

ABSTRACT *The current VDSL performance test [1] puts insufficient requirements on the nature of the noise signals to be used in the performance test. We propose to use a mask for the crest distribution function of the test noise.*

1 Introduction

The current VDSL performance test [1] puts a few requirements on the nature of the noise signals to be used in the performance test:

> "The noise shall be random in nature and Gaussian distributed. The crest factor of the noise source shall be between 5 and 8"

The characterisation above of the amplitude distribution of the noise for performance tests has been argued to be both insufficient and impracticable [2].

The insufficience is mainly related to the fact that the above prescription leaves a lot of ambiguity with respect to the amplitude distribution of the actual impairment noise signal. This has been shown to lead to ambiguous test results [2]. Furthermore, although the need to have high crest factors is widely acknowledged [3,4], the requirement of both gaussian behaviour and of high crestfactor in practice leads to very long noise signals.

This paper proposes to use a mask to be applied as a the lower bound to the Crest Distribution Function of the *actual impairment noise* to be used in the VDSL performance test. The definition of such a mask defines the amplitude characteristics of the impairment noise much better than the current definition, while at the same time allowing for realistically short noise signals. **However, the details of the mask are open to further study: it still has to be verified experimentally that the detailed mask proposed in this paper is sufficiently tight to unambiguously define the performance test.**

This paper is organised as follows. Section 2 defines the crest distribution function (CDF), and proposes a lower limit mask to be applied to the measured CDF of the impairment noise in the VDSL performance test. Section 3 discusses the details of the proposal. In particular, it shows that realistically short signals can be constructed that obey the proposed mask. The appendix contains the details of a method to change the CDF of a given signal to a desired shape.

2 Proposed mask

For a signal u(t), the crest factor of this signal is defined as $CF = |u_{peak}| / u_{RMS}$. The details of the amplitude distribution of the signal u(t) are given by the *crest distribution function* (CDF), which is defined as follows: $CDF_u(a)$ is the fraction of the time that the absolute value of the signal divided by its RMS value exceeds a. In formula:

$$CDF_u(a) = \frac{1}{T} m\left(\left\{t \in [0,T] \middle| \; |u(t)| / u_{RMS} > a\right\}\right)$$

The crest distribution function of the impairment noise to be injected at the adding element in shall be bound from below by the following mask:

$$M(a) = (1-e) \cdot \left(1 - erf\left(a / \sqrt{2}\right)\right), \qquad 0 < a < CF \, .$$

NOTE: The proposed lower limit is obtained by demanding that up to values |u|<CF, the measured crest distribution is bounded from below by the true gaussian distribution, up to an accuracy of ε. This will guarantee that the actual noise signal:

1. has a sufficiently high crest factor (viz. larger than CF)
2. is close to an ideal gaussian distribution.

Provisional values for the parameters in this formula are given in Table 1, the mask itself is shown in Figure 2.

Parameter	Value
CF	5
ε	0.05

Table 1: Proposed values for the crest distribution mask.

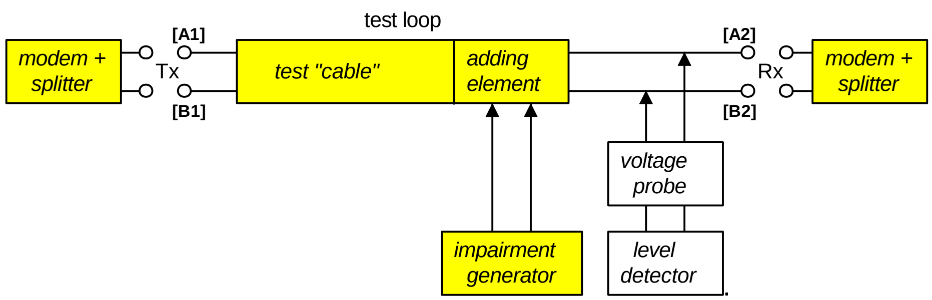

Figure 1: Functional description of the set-up of the performance test.

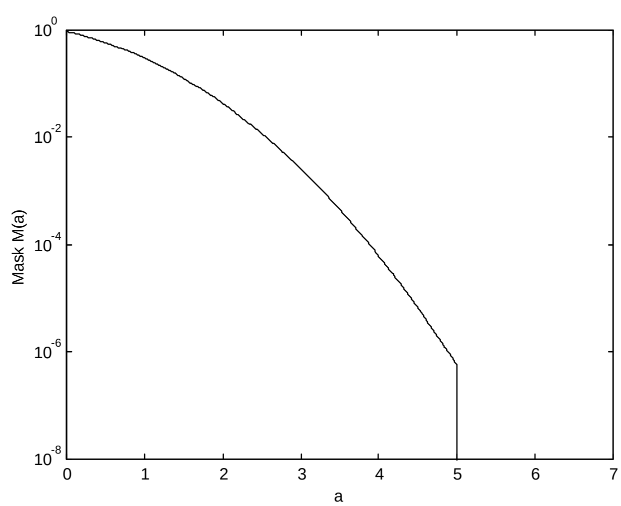

Figure 2: Lower limit mask for the Crest Distribution Function.

3 Discussion

This section elaborates on the proposal described in section 2.

3.1 The Crest Distribution Function

Consider a signal $u(t)$ with $t \in [0,T]$. For this signal the *amplitude distribution* is given by [5]

$$F_u(a) = \frac{1}{T} m\left(\left\{ t \in [0,T] \middle| \; |u(t)| > a \right\}\right).$$

For a stochastic signal of infinite length the *amplitude distribution* is related to the probability density as follows:

$$F_u(a) = P(|u| > a) = 2\int_a^\infty p(x)dx\,,$$

with p(x) dx the probability of finding the value of the signal between x and x+dx.

For a gaussian signal, p(x) is the standard normal distribution function. For a sample of infinite length, this leads to an amplitude distribution given by:

$$F_u(a) = P(|u| > a) = 1 - erf\left(a/\sqrt{2}\right) = 1 - \frac{2}{\sqrt{p}} \int_o^{a/\sqrt{2}} \exp\left(-t^2\right)dt$$

It is usefull to define the amplitude distribution function function of a signal $u(t)$ independent from its RMS value u_{RMS}. To achieve this, the *Crest Distribution Function* (CDF) is defined as

$$CDF_u(a) = F_{u/u_{RMS}}(a)$$

This independence of the CDF on u_{RMS} implies the following ‘power constraint' on the crest distribution factor

$$2\int_0^\infty CDF_u(a)ada = 1\,.$$

3.2 Towards higher crest factors

White gaussian noise can be constructed by taking independent values from a gaussian distribution. For such a finite sample of a noise signal, the expected crest factor depends on the length of the sample. To obtain high crest factors with a finite sample requires a very high length.

Constructing a sample with both a limited length and a high crest factor can be achieved by adjusting the exponential tails of the gaussian probability density is such a way that more statistical weight is moved to the larger values. An example of this is sketched in Figure 3: the probability density depicted there is the sum of a truncated gaussian distribution and a uniform distribution.

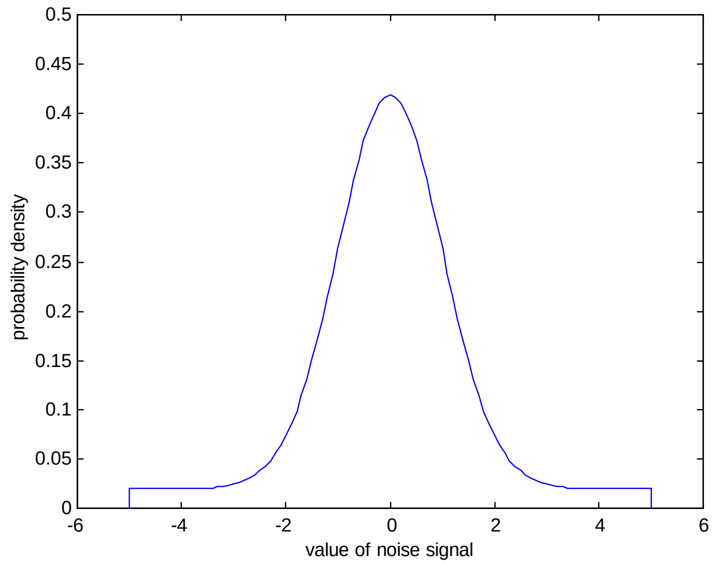

Figure 3: An alternative probability distribution density function. Already for a relatively small number of samples drawn from this distribution, the peak value of the signal is expected to be close to the edges of the distribution.

For both the true gaussian distribution and for the alternative of Figure 3, the CDF (of a sample of infinite length) is sketched in Figure 4.

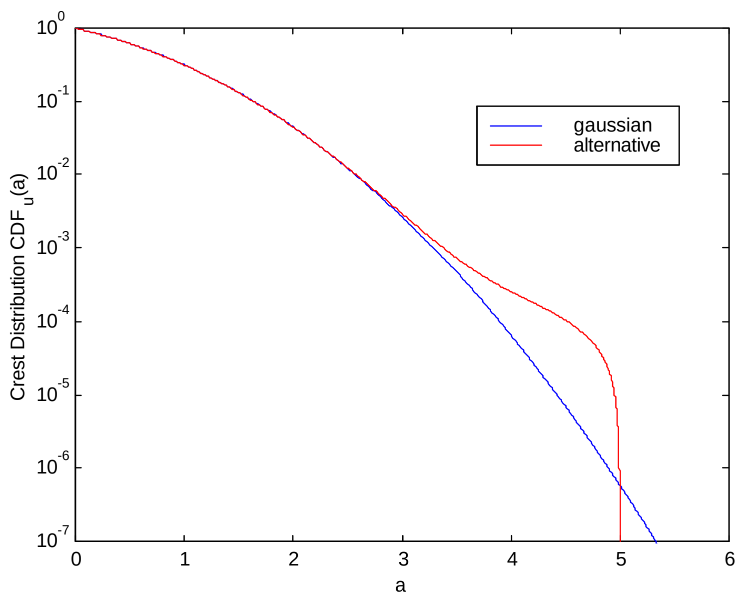

Figure 4: Crest distribution for a sample of infinite length drawn from respectively a gaussian distribution and from the alternative distribution density of Figure 3.

3.3 Mask for the Crest Distribution function

It is the purpose of this paper to specify a lower limit for the measured crest distribution of the actual signal that will be injected on the link under test. This will guarantee that the actual noise signal

1. has a sufficiently high crest factor
2. is close to an ideal gaussian distribution.

This has lead to the prescription of the lower limit mask in the proposal in section 2.

We explicitly do not define an upper limit to the measured CDF.
First of all, the requirement of a tight lower limit (i.e. ε is small) together with the power constraint already puts an effective upper limit on the CDF.
Second, consider a limited noise signal, that is, either the number of points N is limited or, equivalently, the duration T is limited (the effective number of points is then of the order N = T/B, with B the bandwidth of the signal) . For such a signal the measured CDF will have a granularity of 1/N. This means that a requirement like e.g. "$CDF_u(5)$ should be between 1e-8 and 1e-7" cannot possibly be obeyed by a signal of (effective) length N = 1e6.

In Figure 5 three curves are shown: the crest mask as defined above, the crest distribution for a finite sample (100.000 points) of true gaussian noise, and the crest distribution of this same sample of white noise, but with its crest distriibution shaped according to a distribution of the type of Figure 3 (cf. Appendix). Clearly, the crest distribution of the finite gaussian sample (with crest factor of 4.8) does not comply to the mask, whereas the sample is compliant after shaping its CDF (the reshaped sample has crest factor 6). This shows that it is possible to construct signals of practical length that obey lower limit masks on the CDF.

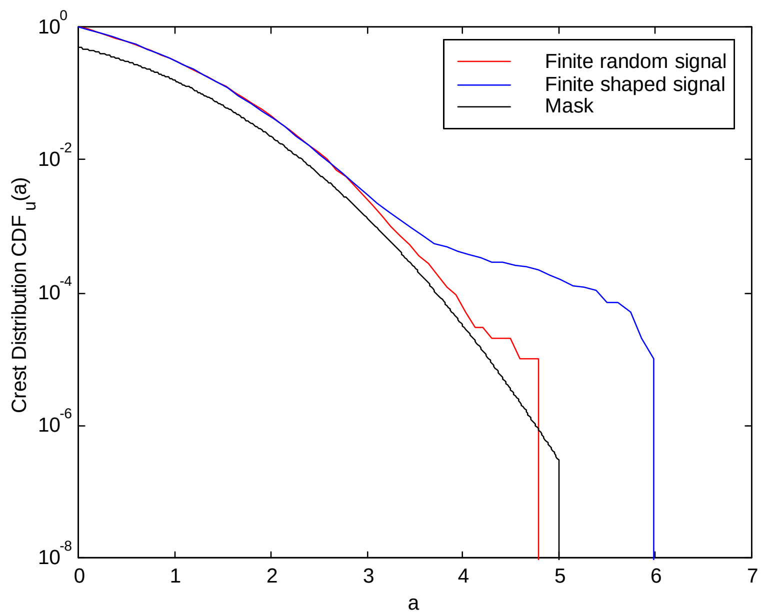

Figure 5: Crest distribution functions. The black curve (we took ε = 0.5 for clarity) denotes the mask. The non-compliant red curve is the crest distribution of a finite length sample (100.000 points) drawn from a standard normal distribution. The compliant blue curve corresponds to the same sample with its crest distribution shaped according to a distribution of the type of Figure 3.

4 References

[1] ETSI TM6, DTS/TM-06003-1(draft) v0.0.7 (1998-2), “Transmission and Multiplexing; Access Transmission on Metallic Access Cables; Very High Speed Digital Subscriber Line (VDSL); Part 1: Functional Requirements”, February 1998.
[2] KPN Research, “PSD + Crest factor is not sufficient to specify noise in performance tests”, ETSI TM6, TD16, June 22nd-26th , Luleå, Sweden..
[3] Schmid Telecom AG, “Proposal for the modification of the HDSL test noise”, ETSI TM6, TD11, April 20th - 24th, Antwerp, Belgium.
[4] P. Nurfluss, A. Kliger, “Proposal to include high crest factor noise in the 2B1Q HDSL performance tests”, ETSI TMD, TD40, January 26th - 30th, 1998, Madrid, Spain.
[5] S. Boyd, “Multitone Signals with Low Crest Factor”, IEEE Transactions on Circuits and Systems, Vol. CAS-33, no. 10, october 1986.

5 Appendix: Algorithm for shaping the CDF of noise signals

The following algorithm can be used to change to CDF of a signal u(t) to any[1] CDF desired.

Let G(a) be the crest distribution of the original signal: G(a) = CDF_u(a). Let H(a) be the desired crest factor distribution function. Then the signal v(t) defined by

$$v(t) = H^{-1}\left(G\left(|u(t)|\right)\right) \cdot \operatorname{sgn}(u(t))$$

has the desired crest distribution function, i.e. CDF_v(a) = H(a).

Proof:
Consider the amplitude distribution function of the signal v(t):

$$\begin{aligned}
F_v(a) &= \frac{1}{T} m\left(\left\{t \in [0,T] \middle| \; |v(t)| > a\right\}\right) \\
&= \frac{1}{T} m\left(\left\{t \in [0,T] \middle| \; H^{-1}\left(G\left(|u(t)|\right)\right) > a\right\}\right) \\
&= \frac{1}{T} m\left(\left\{t \in [0,T] \middle| \; |u(t)| > G^{-1}\left(H(a)\right)\right\}\right) \\
&= F_u\left(G^{-1}\left(H(a)\right)\right) \\
&= G\left(G^{-1}\left(H(a)\right)\right) \\
&= H(a).
\end{aligned}$$

Note that shaping the CDF of a signal is an operation in the time-domain, that might influence the spectral properties of the signal (frequency domain): for noise whose PSD is not flat (i.e. non-white noise), shaping of the CDF will influence the shape of the PSD.

[1] Up to the implicit granularity 1/N defined by the the length of the signal u(t).