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ROBUST CALIBRATION METHODS FOR ONEPORT ACOUSTIC NETWORK ANALYZERS WITH MULTIPLE STANDARD REFLECTORS

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ABSTRACT

An improved mathematical method is here proposed to extract the de-embedding parameters of one-port Acoustic Network Analyzers, equipped with full error correction. Acoustic Network Analyzers are used for calibrated measurements of acoustic reflection, impedance and admittance of objects in regular tubes. The improved calibration method is primarily based on redundancy by using a large number of standard reflectors, and secondly on the spreading of errors in a balanced way. This has made the calibration method robust and insensitive to small errors, as compared to a simple three-point calibration.

The applicability of the instrument has been improved by using different calibration methods for reflection, impedance and admittance measurements. Because absolute impedance values range from zero to infinity, methods are proposed to handle extreme values in case of impedance or admittance de-embedding.

Iterative calculation methods are avoided by linearization of the calibration equations. As a result, the de-embedding parameters are resolved in closed mathematical form. The linearization has been achieved by error weighting, at the cost of a small degradation in accuracy balance.
Therefore, various accuracy aspects are analyzed in detail. First, the error weighting and the influence of error weighting on error spreading is calculated. Next, the influence of the standard reflector accuracy on the overall de-embedding accuracy is evaluated. Finally, the accuracy analysis has resulted in recommendations for different calibration methods associated with different measurement ranges, in case of impedance or admittance de-embedding.

The proposed methods are demonstrated experimentally and compared with a simple three-point calibration.

0. INTRODUCTION

In many practical problems, acoustic properties such as absorption and reflection of porous material, resonating effects in loudspeaker cabinets or the radiation impedance of acoustic sources have to be determined by measurement. In a well-defined measurement setup, the measurement object is embedded in a closed acoustic system and linked to an acoustic driver via regular tubes. If the measuring object has only one connection with the measuring instrument, it is called a one-port.
A classic and wide-used method for reflection measurement is based on the detection of successive minima and maxima of the standing wave pattern in a

tube, terminated with the measuring object (one-port). However, this technique is time consuming since the traversing mechanism is usually operated manually. Recently, a one-port Acoustic Network Analyzer, with direct reading capabilities, has been developed [1] for reflection measurements on guided acoustic waves in regular tubes. Figure 1 shows two examples of one-port measurements with an acoustic Network Analyzer.

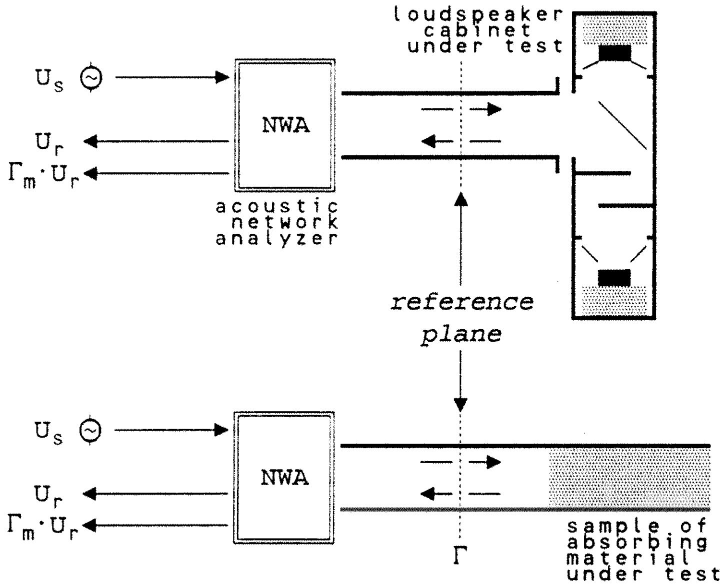

Fig 1 *Two examples of reflection measurements on acoustic objects, using a Network Analyzer. An electrical stimulus signal U_s causes an acoustic wave, and the reflected wave causes the Network Analyzer to generate two electrical signals: U_r and $\Gamma_m \cdot U_r$. The measurement target is the complex reflection coefficient Γ, observed through a well defined reference plane, which is closely related to the magnitude ratio $|\Gamma_m|$ and phase difference $\angle\Gamma_m$ between the two electrical response signals.*

As a consequence of frequency dependent effects in the Network Analyzer and other linear errors, the measured reflection Γ_m will not correspond with the actual reflection Γ, observed through a well-defined reference plane. A complete cancellation of all these linear errors by mechanical improvement is essentially impossible [2], because this requires a Network Analyzer with infinite small dimensions.
A superior improvement method above mechanical improvement is demonstrated in ref. [2] and uses mathematical techniques to correct for all these linear measurement errors. The mathematical error correction is based on a linear error model of the measurement setup which fully characterizes the one-port Network Analyzer with three [2] error correcting parameters.

An essential procedure for mathematical error correction is the extraction of the three error-correcting parameters, which is called calibration. During calibration, various measured reflection values Γ_m are compared with the associated actual reflection values Γ obtained from measurements on well-defined standard reflectors. Based on the difference between the measured values and the actual reflections, the error parameters are reconstructed.

Convenient reflectors for usage as standard reflector are: (offset) Covers, (offset) Opens and ideal Absorbers. An offset Cover or Open is a tube that is covered or left open at its end, at a well defined position. The offset is defined as the distance between the chosen location of the reference

plane and the tube end. An Absorber is something like an open tube, filled with absorbing material.
Covers with zero offset have a reflection coefficient of $\Gamma=1$, which makes them attractive for calibration purposes. Opens with zero offset have a reflection coefficient of $\Gamma\approx-1$, due to minor radiation effects, which makes them second best as choice for standard reflector. Ideal Absorbers have zero reflection by definition, but are difficult to realize with high accuracy.

In ref. [2], a simple calibration method is described, based on measurements of exactly three standard reflectors. Although that calibration method is capable of a drastic improvement of the measurement accuracy and measurement bandwidth, its final accuracy is sensitive to small errors during the calibration.
These calibration errors are caused by distortion, quantization, drift, noise and hum in the Network Analyzer [2], and by the limited accuracy of the standard reflectors. For instance, in case an offset Cover is used as standard reflector, the inaccuracy in sound velocity forms one of the principal accuracy limitations, because it varies with ambient temperature and humidity.

In this article, we propose an improved calibration method, in which the calibration errors are reduced by appropriate averaging techniques. The method uses the raw measurements on more than three standard reflectors and resolves the overdetermined calibration equations in a least-squares sense. As a result, the errors are symmetrically spread out over all measurements and individual errors do not deteriorate the overall accuracy.

1. DE-EMBEDDING EQUATIONS

Figure 2 shows an appropriate linear error model of a one-port Acoustic Network Analyzer for mathematical error correction of linear instrument errors. It is basically an ideal Network Analyzer cascaded with a virtual acoustic two-port. As was demonstrated in ref. [2], all linear instrument errors can be combined and thought of as concentrated in this virtual acoustic two-port.
Although the chosen two-port representation is irrelevant, we will confine ourselves to a normalized representation with scattering parameters [2],[3].

The advantage of the linear error model in figure 2 is that the scattering parameters of the virtual two-port facilitate the derivation of a mathematical relationship between actual and measured reflection coefficients. The mathematical reconstruction of the actual reflection from the measured reflection is based on one of these relationships.
Another advantage is the possibility to position the reference plane on an arbitrary location. In practical situations, the object of measurement is linked with the analyzer by an acoustic wave guide, and this tube affects the measurement result. The use of mathematical error correction facilitates the positioning of the reference plane at any location in this wave guide.

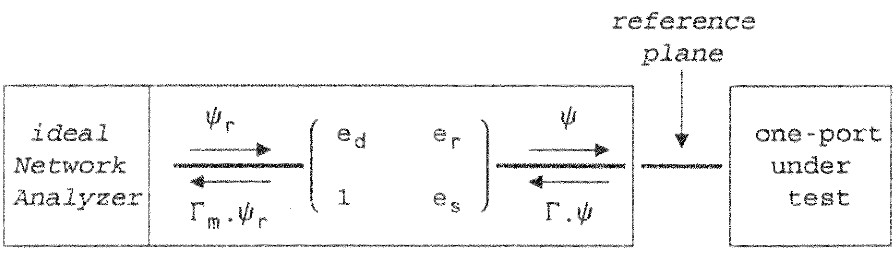

Fig 2 Full linear error model for a one-port Acoustic Network

Analyzer, with normalized error correcting scattering parameters.

The actual reflection Γ through the reference plane is embedded in the wave guide between the virtual error two-port and the object of measurement. When all three scattering parameters $[e_d, e_r, e_s]$ are known, then the actual value of Γ can be reconstructed from the measured value Γ_m.

The reconstruction of a measurement target from raw measurements, such as reflection, impedance or admittance, is called *de-embedding*, and the required set of parameters, such as $[e_d, e_r, e_s]$, are called *de-embedding parameters*, or *error-correcting parameters*.
The measurement and extraction procedure of the de-embedding parameters from measurements on well defined reflectors, is called *calibration*.

In ref. [2] the derivation of a de-embedding equation is described, but this was limited to reflection de-embedding. Although the Acoustic Network Analyzer is primarily intended for reflection de-embedding, a simple transformation of reflection into acoustic impedance Z or acoustic admittance Y will extend the applicability of the instrument. Figure 3 tabulates the de-embedding equations for reflection, impedance and admittance for various sets of de-embedding parameters.

mutual relation	$[e_d, e_r, e_s]$	$[e_d, \Delta_E, e_s]$
$\Gamma = \dfrac{Z - \mathbb{Z}_0}{Z + \mathbb{Z}_0}$	$= \dfrac{(e_d - \Gamma_m)}{e_s \cdot (e_d - \Gamma_m) - e_r}$	$= \dfrac{e_d - \Gamma_m}{\Delta_E - e_s \cdot \Gamma_m}$
$Z/\mathbb{Z}_0 = \dfrac{1 + \Gamma}{1 - \Gamma}$	$= \dfrac{(e_s+1) \cdot (e_d - \Gamma_m) - e_r}{(e_s-1) \cdot (e_d - \Gamma_m) - e_r}$	$= \dfrac{(\Delta_E + e_d) - (e_s+1) \cdot \Gamma_m}{(\Delta_E - e_d) - (e_s-1) \cdot \Gamma_m}$
$Y \cdot \mathbb{Z}_0 = \dfrac{1 - \Gamma}{1 + \Gamma}$	$= \dfrac{(e_s-1) \cdot (e_d - \Gamma_m) - e_r}{(e_s+1) \cdot (e_d - \Gamma_m) - e_r}$	$= \dfrac{(\Delta_E - e_d) - (e_s-1) \cdot \Gamma_m}{(\Delta_E + e_d) - (e_s+1) \cdot \Gamma_m}$

Fig 3 *Basic de-embedding equations of the actual reflection (Γ), impedance (Z) or admittance (Y) from the measured reflection Γ_m, with $[e_d, e_r, e_s]$ or $[e_d, \Delta_E, e_s]$ as de-embedding parameters.*
e_d = *directivity parameter*
e_s = *source match parameter*
e_r = *reflection tracking parameter*
Δ_E = *matrix determinant* $(e_d e_s - e_r)$

2. ROBUST CALIBRATION WITH MULTIPLE STANDARDS

The calibration for the reconstruction of three de-embedding parameters, can be performed in various ways but always requires at least three independent measurements. One of these methods was described in detail in ref. [2]. Although the reflection of many Covers, Opens and Absorbers were measured, the described method used only three Covers per frequency. The lack of redundancy has prevented this calibration method from being robust, and causes the result to be sensitive to small errors.

The overall accuracy of the calibration is limited by the accuracy of the standard reflector models, and by distortion, drift, noise and hum in the Network Analyzer. For example, a small error in the measurement of an offset distance will cause an error in the reflection phase which increases with the frequency. A robust solution to this problem, that reduces the sensitivity to small calibration errors, is the use of a large number of standard reflectors and an appropriate averaging over all these measurements.

In the case of three standard reflectors, the reconstructed values of the error correcting parameters are unique, and independent of the used calculation method. When these values are used for reflection de-embedding, then the error is forced to zero for these three reflectors, at the cost of decreasing the de-embedding accuracy for other reflectors.
In an overdetermined calibration, the concept *"best estimation"* is closely related to the measurement target: Γ, Z or Y. All calibration errors are then spread out over all calibration samples in a way that minimizes the average de-embedding error.

2.1 NEAR OPTIMAL CALIBRATION

Assume that T represents the actual value of measurement targets such as reflection, impedance or admittance, and that the numbers $[q_1, q_2, q_3]$ represent a set of associated de-embedding parameters. The de-embedding function should provide an accurate prediction of T, based on the measured reflection Γ_m, but predicts a value $(T+\partial T)$.
The overview in figure 3 demonstrates that the de-embedding function can be transformed into a bilinear function, and thus that the relationship between measured value and de-embedded value has the following general form:

$$T + \partial T = \frac{q_1 - q_2 \cdot \Gamma_m}{1 - q_3 \cdot \Gamma_m}$$

$$q_1 - q_2 \cdot \Gamma_m + q_3 \cdot T \cdot \Gamma_m = T + \partial T \cdot (1 - q_3 \cdot \Gamma_m)$$

An optimal calibration requires that the average of absolute (∂T) or relative $(\partial T/T)$ errors, associated with the standard reflectors, is minimized in a least-squares sense.
Unfortunately, when a set of four or more simultaneous equations of this non-linear form has to be resolved in a least-squares sense, then the solution that minimizes the average error must be found in an iterative way.
A more convenient approach, that is superior in simplicity, will result in a slightly different solution close to this optimum. It is based on the optimization of a weighted error, of which the weighting factor is chosen to facilitate the linearization of the optimization equation.
We define the following concepts:

absolute error:	∂T
relative error:	$\partial T/T$
weighting factor:	$W = (1 - q_3 \cdot \Gamma_m)$
weighted error:	$W \cdot \partial T = (1 - q_3 \cdot \Gamma_m) \cdot \partial T$

This approach minimizes the weighted error instead of the absolute error, and is therefore called a *near-optimal* calibration. In some cases, the weighted error is close to the *absolute* error (∂T), and in other cases it is close to the *relative* error $(\partial T/T)$. This is worked out in section 3.
The use of a modified optimization goal, by the use of error weighting, will reduce the complexity of the equation to the following linear form:

$$q_1 - q_2 \cdot \Gamma_m + q_3 \cdot T \cdot \Gamma_m = T + W \cdot \partial T$$

Owing to this linear form, each calibration sample contributes to a row of the following matrix equation:

$$\begin{pmatrix} 1 & -\Gamma_{m1} & T_1\cdot\Gamma_{m1} \\ 1 & -\Gamma_{m2} & T_2\cdot\Gamma_{m2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} * \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} T_1 + W_1\cdot\partial T_1 \\ T_2 + W_2\cdot\partial T_2 \\ \vdots \\ \vdots \end{pmatrix}$$

In the case of three standard reflectors, the solution for the de-embedding parameters is unique. When the absolute de-embedding error for these reflectors is defined as $\partial T=0$, then these parameters $[q_1, q_2, q_3]$ are found by matrix inversion followed by matrix multiplication. In the case of four or more reflectors, an exact solution, that fits all actual values T with zero error, does not exist in general.
A near-optimal solution minimizes the power average of the weighted errors $\langle |W_i\cdot\partial T_i|^2\rangle$ in this overdetermined matrix equation. The least-squares solution of a set with linear equations is unique and can be found by an overdetermined left-hand matrix division. This is described in detail in section 4. Using the Matlab conventions for matrix division [5], the least-squares solution has the following closed mathematical form:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 1 & -\Gamma_{m1} & T_1\cdot\Gamma_{m1} \\ 1 & -\Gamma_{m2} & T_2\cdot\Gamma_{m2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} \backslash \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ \vdots \end{pmatrix}$$

When $[q_1, q_2, q_3]$ and $T(\Gamma_m)$ are substituted by the formulas summarized in figure 3, then the left-hand matrix division results in the near-optimal calibration solution for reflection, impedance and admittance. In figure 3, these solutions are tabulated.

near-optimal reflection calibration
that minimizes: $\langle |W_\Gamma\cdot\partial\Gamma|^2\rangle \approx \langle |\partial\Gamma|^2\rangle$

$$\begin{pmatrix} e_d/\Delta_E \\ 1/\Delta_E \\ e_s/\Delta_E \end{pmatrix} = \begin{pmatrix} 1 & -\Gamma_{m1} & \Gamma_1\cdot\Gamma_{m1} \\ 1 & -\Gamma_{m2} & \Gamma_2\cdot\Gamma_{m2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} \backslash \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \vdots \end{pmatrix}$$

near-optimal impedance calibration
that minimizes: $\langle |W_Z\cdot\partial Z|^2\rangle \approx \langle |\partial Z\cdot(1-\Gamma)|^2\rangle$

$$\begin{pmatrix} (\Delta_E+e_d)/(\Delta_E-e_d) \\ (e_s+1)/(\Delta_E-e_d) \\ (e_s-1)/(\Delta_E-e_d) \end{pmatrix} = \begin{pmatrix} 1 & -\Gamma_{m1} & Z_1/Z_0\cdot\Gamma_{m1} \\ 1 & -\Gamma_{m2} & Z_2/Z_0\cdot\Gamma_{m2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} \backslash \begin{pmatrix} Z_1/Z_0 \\ Z_2/Z_0 \\ \vdots \\ \vdots \end{pmatrix}$$

near-optimal admittance calibration
that minimizes: $\langle |W_Y\cdot\partial Y|^2\rangle \approx \langle |\partial Y\cdot(1+\Gamma)|^2\rangle$

$$\begin{pmatrix} (\Delta_E-e_d)/(\Delta_E+e_d) \\ (e_s-1)/(\Delta_E+e_d) \\ (e_s+1)/(\Delta_E+e_d) \end{pmatrix} = \begin{pmatrix} 1 & -\Gamma_{m1} & Z_0/Z_1\cdot\Gamma_{m1} \\ 1 & -\Gamma_{m2} & Z_0/Z_2\cdot\Gamma_{m2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix} \backslash \begin{pmatrix} Z_0/Z_1 \\ Z_0/Z_2 \\ \vdots \\ \vdots \end{pmatrix}$$

FIG 4 *An overview of near-optimal calibration solutions in closed mathematical form. The three requested de-embedding parameters* $[e_d, \Delta_e, e_s]$ *or* $[e_d, e_r, e_s]$ *can be obtained from the* *three elements of the solution vector by simple algebraic manipulations.*

2.2 EXPERIMENTAL VERIFICATION

To demonstrate the improvement in accuracy, the data set that was measured in ref. [2] was used to compare a simple three-point calibration with the proposed robust multi-point calibration. The data set contained raw reflection measurement results on 14 offset Covers, 3 offset Opens and 2 offset Absorbers. The offset distance was varied in arbitrary steps from 0 to 2.5 m, and the frequency was varied in 55 logarithmic steps from 30 to 750 Hz.

Although an Open is not preferred as a *primary* calibration standard [2], all offset Opens were used as standard reflector in the robust calibration. The deviation from an ideal offset Open was modeled by an elongation of 22 mm of the offset tube, terminated with an ideal Open. All remaining errors are assumed to be within the measurement accuracy of the used Network Analyzer.

In theory, the de-embedded reflection magnitude of the Covers should have the value $|\Gamma_c|=1$, and of the Opens $|\Gamma_o|\approx1$. In practical situations, a small deviation from the theoretical value will occur, caused by distortion and quantization in the Network Analyzer [2]. Figure 5 shows the magnitude of the de-embedded reflections with the simple three-point calibration that was described in ref. [2]. Figure 6 shows the de-embedded reflection magnitude of the same data set with robust near-optimal reflection calibration, based on all Covers and Opens.

The robust calibration has performed a significant improvement of the overall de-embedding accuracy over the full frequency range. In spite of more than 70% directivity (e_d), the method has even improved the de-embedding above 300 Hz. The absolute reflection error $\partial\Gamma$ of the de-embedded reflection from its theoretical value is shown in figure 7. The 1.5% rms averaged value of this error below 200 Hz is in agreement with the observed non-linear distortion in the Network Analyzer [2] and random effects caused by hum. For a few reflectors, this deviation was significantly higher than the rms-value, which is plotted as max-error.

Fig 5 *Reflection magnitude of various de-embedded offset Covers, Opens and Absorbers, in case a simple three-point calibration was performed with three offset Covers.*

Fig 6 *De-embedded reflection magnitude of the same data set that is used in figure 4, in case a robust near-optimal reflection calibration was performed with all offset Covers.*

Fig 7 *The deviation between de-embedded reflection and actual reflection, for all 14 Covers and 3 Opens. The rms-average of these errors is defined as* $\sqrt{\langle|\partial\Gamma|^2\rangle}$, *and the max error as* $|\partial\Gamma|_{max}$.

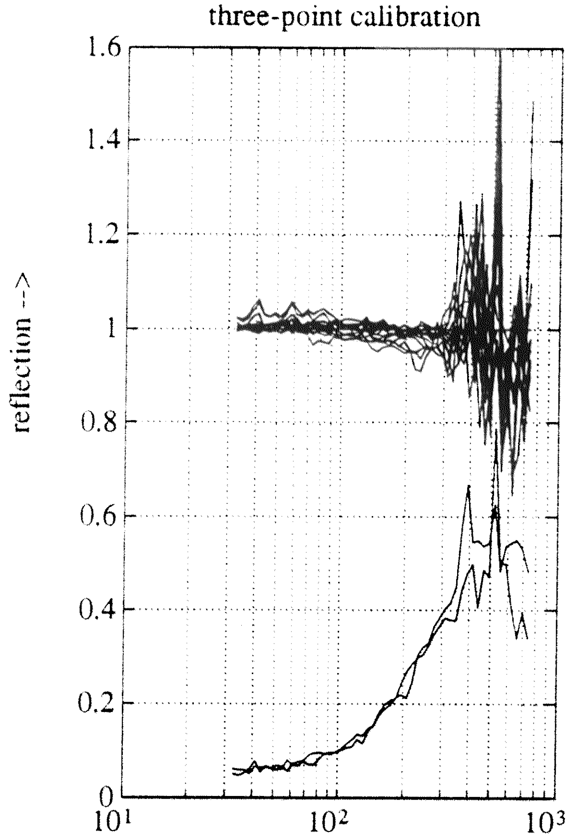

FIG 5. freq [Hz] -->

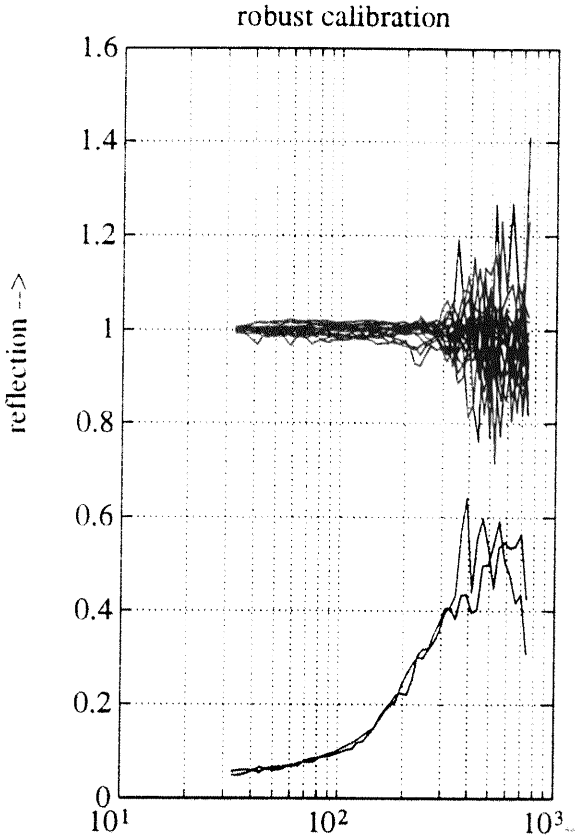

FIG 6. freq [Hz] -->

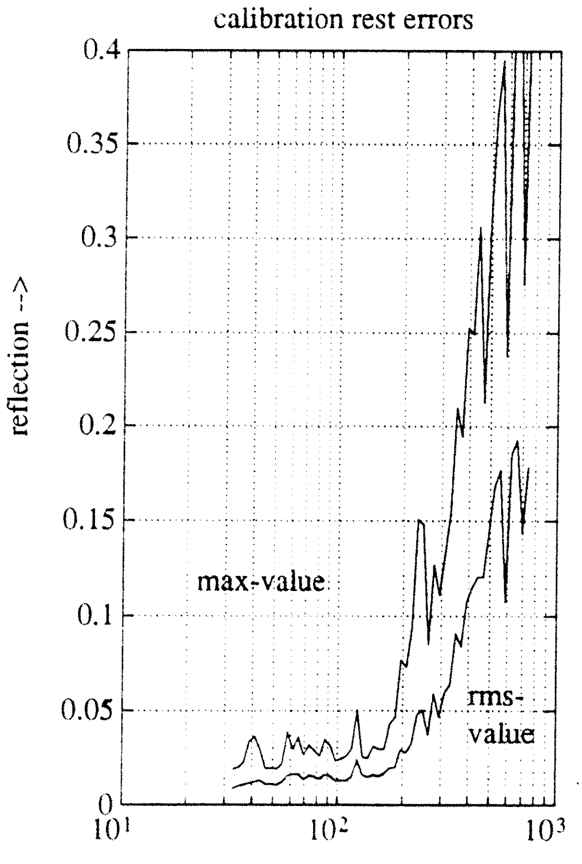

FIG 7. freq [Hz] -->