CHARACTERIZATION AND MODELING OF TRANSISTORS BY VIRTUAL (CIRCUIT) ELEMENTS

ROB F.M. VAN DEN BRINK – 1993

A novel way to specify transistors from linear two-port measurements is proposed, which is based on the transformation of measured matrix parameters into virtual (circuit) elements. It will be demonstrated that fundamental transistor parameters can easily be obtained directly from the virtual element plots.

INTRODUCTION. Two-port measurements on transistors by a network analyzer usually result in a table with S-parameters, and this may have stimulated manufacturers to specify their components by S-parameters in tabular or graphical form. In some circuit simulators these tables can be mixed with model descriptions of other components for circuit analysis.

Although linear devices are fully specified in this way, the relation between S-parameters and fundamental transistor parameters is complicated; these parameters are close related to transistor models. In circuit design the use of models is preferred to data files because data files are restricted to application in circuit analysis only, while models can be applied to both synthesis and analysis.

A novel alternative way for transistor specification and parameter extraction is proposed here, which is based on the transformation of S-parameters into virtual (circuit) elements. To the author's knowledge, the generalization of this mathematical approach is new.

THE METHOD. Fully-calibrated linear two-port measurements on transistors result in 4 complex frequency-dependent parameters, that completely specify the component for all linear purposes. It does not matter whether these 4 parameters are represented as S-, H-, Z-, Y-, Chain- or ABCD-parameters because all these two-port matrix representations can be converted into each other. [1]

This principle of equivalence is not restricted to the mentioned matrix parameters only. Any electronic circuit model, constructed with exactly 4 independent elements, is able to represent a given two-port completely; even when the used impedances and controlled sources do not have any physical relation with the actual device. Some examples are given in figure 1.

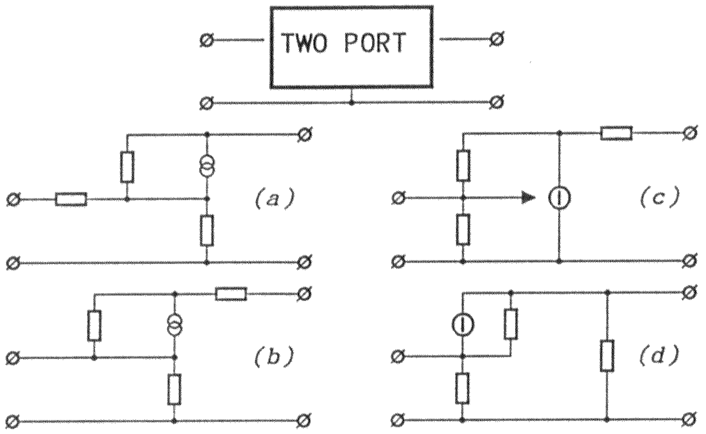


Fig 1 Some arbitrary examples of two-port representations by virtual circuit elements.

A unique solution for all the above virtual (circuit) element values, for example a bipolar transistor, can be obtained from the measured S-parameters. However, it is not guaranteed that these virtual elements correspond to simple electronic elements over a wide frequency interval. Measurements on transistors (figure 4 and 5) show that at least the virtual circuits in figure 2 for BJTs (Bipolar Junction Transistors) and FETs (Field Effect Transistors) meet this requirement.
Both the T- as the Π-model in figure 2 form the basis for commonly used models for BJTs [2][3], while FET models are commonly based on the Π-model only [2][4].

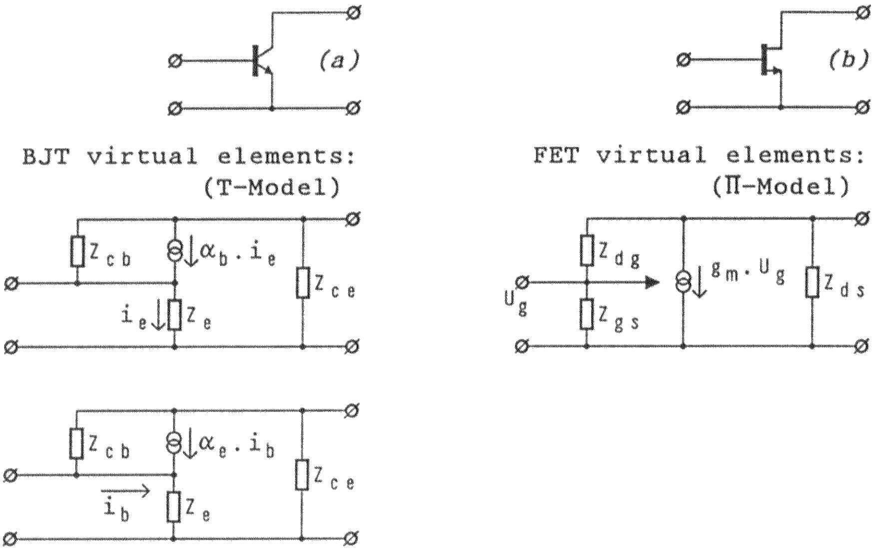


Fig 2 Examples of virtual circuits for transistors of which the virtual elements can be represented by simple electronic components.

The virtual BJT elements $(\alpha_b, z_e, z_{ce}, z_{cb})$ or $(\alpha_e, z_e, z_{ce}, z_{cb})$ and the virtual FET elements $(g_m, z_{gs}, z_{ds}, z_{dg})$ can be obtained from the S-parameters as follows:

$$\Delta s = s_{11} \cdot s_{22} - s_{21} \cdot s_{12}$$
$$s_{0Y} = (1 + s_{11} + s_{22} + \Delta s) \cdot z_0$$

$$Y_{11} = (1 - s_{11} + s_{22} - \Delta s) / s_{0Y}$$
$$Y_{21} = (\quad -2 \cdot s_{21} \quad) / s_{0Y}$$
$$Y_{12} = (\quad -2 \cdot s_{12} \quad) / s_{0Y}$$
$$Y_{22} = (1 + s_{11} - s_{22} - \Delta s) / s_{0Y}$$

for a BJT:

$$z_e = 1 / (Y_{11} + Y_{21})$$
$$z_{ce} = 1 / (Y_{22} + Y_{12})$$
$$z_{cb} = 1 / (\quad -Y_{12})$$
$$\alpha_b = (Y_{21} - Y_{12}) / (Y_{11} + Y_{21})$$

if α_e is preferred to α_b:

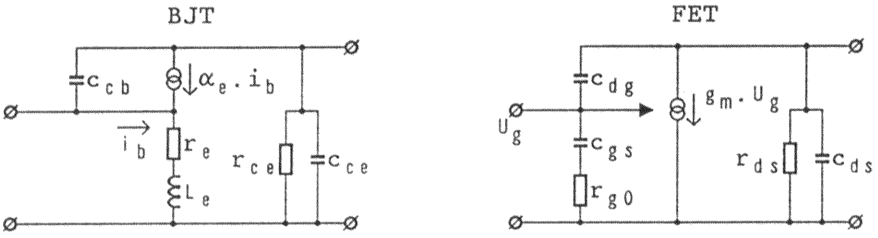
$$\alpha_e = \alpha_b / (1 - \alpha_b)$$
$$\alpha_e = (Y_{21} - Y_{12}) / (Y_{11} + Y_{12})$$

for an FET:

$$g_m = (Y_{21} - Y_{12})$$
$$z_{ds} = 1 / (Y_{22} + Y_{12})$$
$$z_{dg} = 1 / (\quad -Y_{12})$$
$$z_{gs} = 1 / (Y_{11} + Y_{12})$$

Figure 4 and 5 show plots of the virtual elements (outlined in figure 2) of commercially available transistors.

TRANSISTOR PARAMETER EXTRACTION. The virtual elements in figure 2, 4 and 5 can be represented by electronic components over a wide frequency interval. In figure 3 it is shown that each virtual element can be replaced by one or two electrical components; each virtual element is then represented by one or two real parameters.



$$\alpha_b(\omega) = \alpha_{b0} \cdot e^{-j\omega\tau_i}$$

$$\alpha_e(\omega) = \alpha_b/(1-\alpha_b)$$

$$\omega_T = 1/\tau_i$$

$$g_m(\omega) = g_{m0} \cdot e^{-j\omega\tau_i}$$

$$\omega_T = g_{m0}/(c_{gs}+c_{dg})$$

fig 3 Two transistor models which are closely related to the virtual circuits in figure 2.

By transforming the measured S-parameters into virtual elements, the complete modeling problem of obtaining 7 or more parameter values simultaneously, has now been reduced to 4 simple independent modeling problems.

How closely the models in figure 3 match the measured data is shown in figure 4 and 5 by an overlay of the virtual element values of both the model and the measured data. The representation of the virtual elements of both the BFR92a and the CF910 by the models in figure 3 is adequate in a wide frequency interval, except for z_{ce} at low frequencies. Presumably, a more sophisticated model may account for these effects, but this is beyond the scope of this letter. Yet, this discrepancy is restricted to the r_{ce} resistance of z_{ce} only and does not influence the match to the other virtual element values.

Owing to the fact that the simple models in figure 3 result in a very close match of the virtual elements values, the most important linear transistor parameters can easily be extracted from virtual element plots (even manually, without iterations).

CONCLUSIONS. The use of virtual elements instead of matrix parameters reduces modeling of transistors from one overall modeling problem into four simple independent modeling problems. Fundamental device characteristics can therefore be obtained directly from the appropriate virtual element plots, while the device maintains fully specified. This attractive method may encourage manufacturers to specify their linear transistor data in this almost circuit form.

In many cases the match between virtual circuit elements and measured data is close enough and will make sophisticated iterative device modeling superfluous. When the parameters of sophisticated models must be extracted the proposed method can provide very good starting values for automated iterative modeling, and may serve as the preferred optimization goal in optimizers.

In conclusion, the use of virtual elements provide the designer the tools for fast and easy extraction of fundamental transistor parameters

R.F.M. van den BRINK
PTT-Research, Neher Laboratories
P.O.box 421
2260 AK Leidschendam
The Netherlands.

[1] GONZALEZ, G: "Microwave transistor amplifiers" *Prentice-Hall inc., Englewood Cliffs, 1984, page 24-25*
[2] SHEPARD, K and SCHUMACHER, H: "Scaling n NPN and PNP heterostructure bipolar transistors", *El. Letters 1988, vol 24, no 2, page 111-112*
[3] NORDHOLT, E.H: "Design of high-performance negative-feedback amplifiers", *Elsevier, Amsterdam, 1983, page 53*
[4] DAMBRINE, G et al: "A new method for determining the FET small-signal equivalent circuit", *IEEE Trans. on Microwave Theory and Techn. vol 36, no 7, july 1988, p1151-1159.*

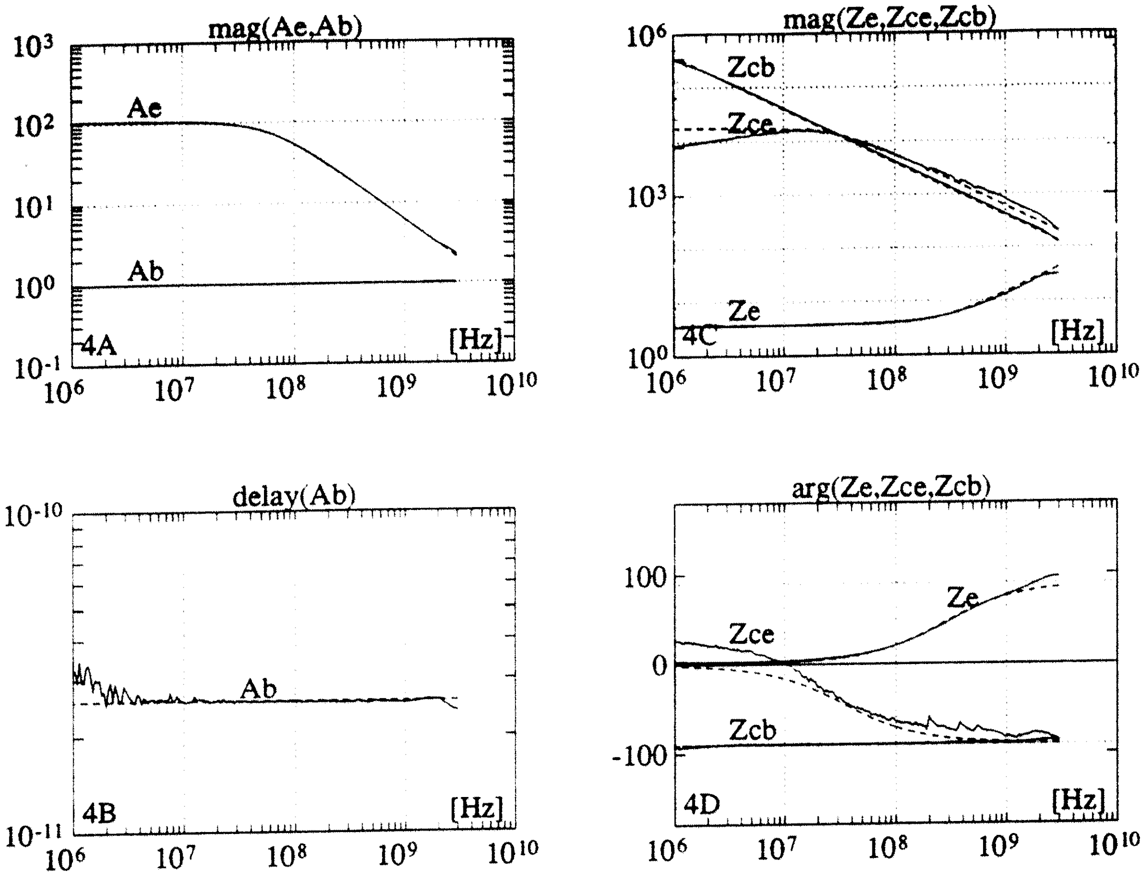


fig 4 Virtual element plots of a BFR92a bipolar junction transitor, based on the T-model in figure 2 (I_c = 10 mA).

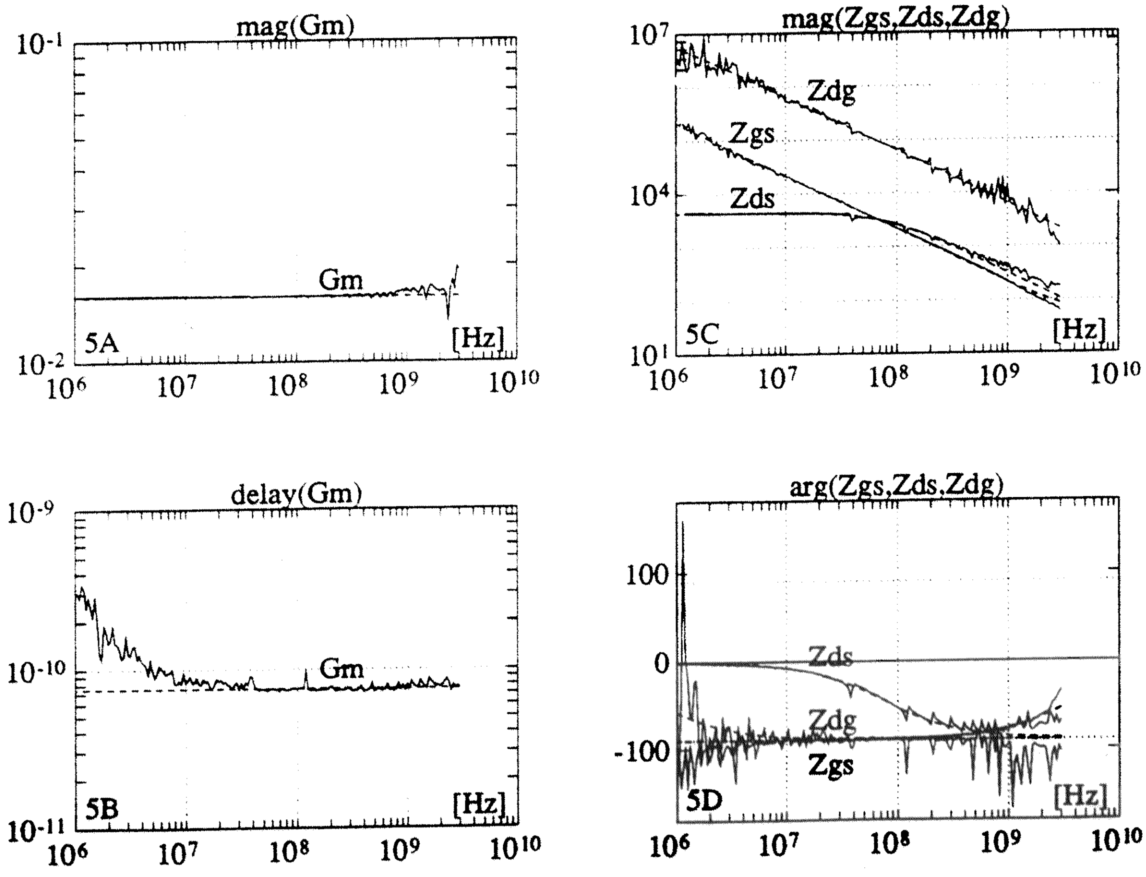


fig 5 Virtual element plots of a CF910 Field Effect transitor, based on the Π-model in figure 2 (I_d = 10 mA).