Theory of Lightwave Synthetic Noise Generation

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***Abstract*— Lightwave synthetic noise generators are instruments for microwave-photonic measurements [1]–[4]. They facilitate the measurement of noise and bandwidth in lightwave receivers and electrical amplifiers. When synthetic noise is generated at the input of a lightwave receiver, the equivalent input noise of the receiver can easily be determined by comparing its unknown level with the known level of the synthetic noise. This report demonstrates that elementary properties of synthetic noise can be derived from a simple theoretical model. These properties include: 1) the difference between natural noise and synthetic noise, 2) requirements for reliable operation, and 3) spectral aspects.**

I. Introduction

A LIGHTWAVE source, generating white electrical noise in p-i-n photodiodes, is of particular interest for noise (and bandwidth) measurements. When synthetic noise is generated at the input of a lightwave receiver, the equivalent input noise of the receiver can easily be determined by comparing its unknown level with a known level of synthetic noise. The feasibility of synthetic noise generators have been demonstrated in [1], [3], and measurement applications have been demonstrated for lightwave receiver bandwidth [1] and noise [2], [4], [5]. Previously reported theory was restricted to a simple approximation of synthetic noise spectra [1], a marginal discussion on incoherence requirements (level, spectrum) [3], and no discussion on the non-Gaussian nature of synthetic noise. This paper will address these issues.

The lightwave synthetic noise generator discussed here is basically an FM modulated laser, cascaded by an interferometer with unequal arms. Illumination of a p-i-n photodiode results in an (in)coherent mix of lightwave frequencies. The spectrum of the photocurrent is essentially a discrete FM spectrum, but is semi-random in nature due to laser linewidth and external noise injection [3]. The spectrum of this photo current is virtually white up to an upper frequency. This upper limit is user-definable and adjustable from several MHz to several GHz or more.

This report is restricted to the theoretical aspects of synthetic noise, which will be examined from three different viewpoints: time domain characteristics, incoherence requirements, and frequency domain characteristics. It discusses the difference in statistical properties between synthetic noise and natural noise, evaluates the minimum level of noise injection, and calculates the spectrum for arbitrary modulation signals.

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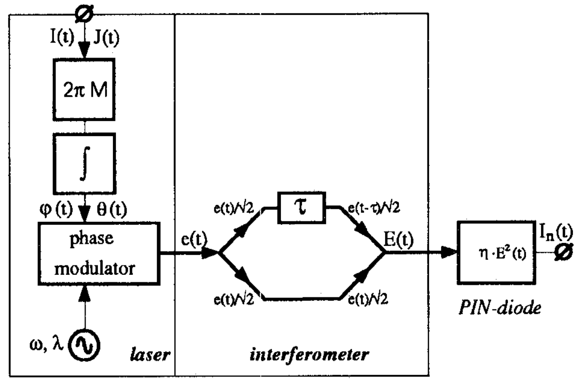

Fig. 1. A calculation model of a lightwave synthetic noise generator, illuminating a p-i-n photodiode. Signal $I(t) + J(t)$ represents the sum of a periodical and a random modulation signal, M the FM sensitivity of the laser, τ the differential delay of the interferometer, η the sensitivity of the p-i-n photo diode, and $e(t)$, $E(t)$ various laser fields.

II. Time Domain Analysis

The lightwave frequency of a synthetic noise generator is electrically FM modulated, split, (partly) delayed in an interferometer, and finally recombined. This results in a composition of two lightwave FM signals, each with different *momentary* frequency. When this composite signal illuminates a p-i-n photodiode, as shown in Fig. 1, the associated photocurrent is random in nature due to an incoherent mix of the two lightwave frequencies in the composite signal. Section II-A describes the calculation model that is evaluated in Section II-B, to support the analysis of synthetic noise in the time domain.

A. Calculation Model

Fig. 1 shows a calculation model of the generator in which the FM modulation process is represented by an integrator and a phase modulator. Similar models are commonly used in FM calculation methods associated with laser linewidth [6]–[9] or interferometric measurement setups [1] and [10]–[14].

The validation of this model is restricted to single-mode lasers, having an FM modulation factor M that is constant over a wide frequency range. This holds for multisection DBR lasers with separated FM modulation input. Direct modulated (single section) lasers suffer from parasitic IM effects and a frequency dependent modulation factor M due to thermal/carrier induced FM effects.

Our analysis assumes that the laser diode is biased properly above threshold and that the light beams in both arms of the interferometer are adjusted to be equally polarized.

The composed modulation signal $I(t) + J(t)$ represents the sum of a periodical and a random signal. The random signal $J(t)$ is essential for generating noise, while the periodic signal $I(t)$ is essential for spreading this noise out over a wide frequency band.

Signal $I(t)$ is fully externally generated. Signal $J(t)$ includes the representation of all parasitic FM effects, such as laser linewidth. When these random effects are too weak, an optional random signal can be injected to increase the spectral flatness of the synthetic noise [3]. Random IM-effects, such as laser intensity noise (RIN), are ignored.

Another random effect that is ignored is the mode competition in lasers that are not perfectly single-mode. The application of multimode lasers in synthetic noise generators is dissuaded. They will degrade the controllability of synthetic noise generation, which facilitates a uniform concentration of noise power in a user-definable bandwidth [3].

B. Calculation of Photocurrent

This section derives the relation between the synthetic noise current $I_n(t)$ generated in the photo diode and the modulation signal $I(t) + J(t)$. Let $\varphi(t) + \theta(t)$ be the modulation phase at the input of the phase modulator, and τ the differential delay time in the interferometer, then the following expressions hold

$$\varphi(t) \stackrel{\text{def}}{=} 2\pi \cdot M \cdot \int_{-\infty}^{t} I(t) \cdot dt = \text{(periodic)} \tag{1a}$$

$$\theta(t) \stackrel{\text{def}}{=} 2\pi \cdot M \cdot \int_{-\infty}^{t} J(t) \cdot dt = \text{(random)} \tag{1b}$$

$$\Delta\varphi(t, \tau) \stackrel{\text{def}}{=} \varphi(t) - \varphi(t-\tau) = 2\pi \cdot M \cdot \int_{t-\tau}^{t} I(t) \cdot dt \tag{2a}$$

$$\Delta\theta(t, \tau) \stackrel{\text{def}}{=} \theta(t) - \theta(t-\tau) = 2\pi \cdot M \cdot \int_{t-\tau}^{t} J(t) \cdot dt. \tag{2b}$$

Let $\alpha_r(t)$ and $\alpha_d(t)$ be the momentary phase of the modulated lightwave signals $e(t)$ and $e(t-\tau)$, and E_0 their magnitude then the composite lightwave signal $E(t)$ is equal to

$$\begin{aligned} E^2(t) &= [\sqrt{\tfrac{1}{2}} \cdot e(t) + \sqrt{\tfrac{1}{2}} \cdot e(t-\tau)]^2 \\ &= \tfrac{1}{2} \cdot E_0^2 \cdot \{\cos(\alpha_r) + \cos(\alpha_d)\}^2 \\ &= \tfrac{1}{2} \cdot E_0^2 \cdot \{1 + \cos(\alpha_r - \alpha_d) + \cos(\alpha_r + \alpha_d) \\ &\quad + \tfrac{1}{2} \cdot \cos(2 \cdot \alpha_r) + \tfrac{1}{2} \cdot \cos(2 \cdot \alpha_d)\}. \end{aligned} \tag{3}$$

Illuminating a p-i-n photo diode with this composite signal results in a photo current $I_n(t)$ proportional to $E^2(t)$. All currents at lightwave frequencies are filtered, which results in

$$I_n(t) = (\tfrac{1}{2} \cdot \eta E_0^2) \cdot \{1 + \cos(\alpha_r - \alpha_d) + 0\}$$

$$I_n(t) \stackrel{\text{def}}{=} I_{n0} \cdot \{1 + \cos[\Delta\alpha(t, \tau)]\} \tag{4a}$$

$$\Delta\alpha(t, \tau) = 2\pi \cdot M \cdot \int_{t-\tau}^{t} [I(t) + J(t)] \cdot dt + \omega_0 \cdot \tau. \tag{4b}$$

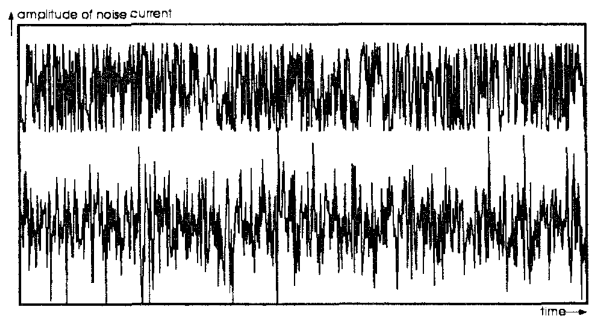

Fig. 2. Simulated oscilloscope view of synthetic noise (upper trace) and natural noise (lower trace). The rms-values of both noisy signals are equal but they are different in nature. Synthetic noise has well-defined upper and lower magnitude limits, while natural noise is virtually unlimited. This latter is a consequence of its Gaussian distribution, which means a low, but nonzero, probability of large spikes.

The differential phase $\Delta\alpha(t, \tau) = \Delta\varphi(t, \tau) + \Delta\theta(t, \tau) + \omega_0 \cdot \tau$ is a rapid fluctuating quantity, due to laser linewidth (phase noise), thermal drift, and noise injection [3]. The origin of all these fluctuations is represented by modulation signal $J(t)$.

The constant offset $\omega_0 \cdot \tau$ in $\Delta\alpha$ is of minor importance for the statistical properties of $\cos(\Delta\alpha)$ and I_n when the rms-fluctuation of $\Delta\alpha$ is significantly higher than π. This condition is a basic requirement for a "well-designed" synthetic-noise generator, as discussed in Section III and illustrated in Fig. 4.

C. Simulated Oscilloscope View

In practice, the photo current $I_n(t)$ changes rapidly and at random. When the rms-level of random modulation signal $J(t)$ is high enough, then current $I_n(t)$ varies from zero to twice the mean photo current I_{n0}. The upper trace of Fig. 2 shows a simulated oscilloscope view of this photo current. Signal $J(t)$ was simulated by Gaussian distributed random numbers, and signal $I(t)$ was set to zero.

The synthetic noise in the upper trace has a random appearance, but well-defined upper and lower magnitude limits. The lower trace represents natural noise, using Gaussian (normal) distributed numbers. It demonstrates that the statistical distribution of synthetic noise is significantly different, which is of importance for some measurement applications of the generator. Section II-D discusses these differences and Section II-E their consequences when measuring noise.

D. Probability Distribution

The magnitude distribution of the synthetic noise current $I_n(t)$ is (nearly) independent of the magnitude distribution of the random modulation signal $J(t)$ when the rms-magnitude of $\Delta\alpha$ is large ($\gg\pi$). This is because the function $\cos(\Delta\alpha)$ wraps the results in 2π intervals, which causes the signal $\varphi = \arccos[\cos(\Delta\alpha)]$ to be random with near *uniform* distribution. In the limit that $\Delta\alpha_{\text{rms}} \to \infty$, the distribution of $\arccos[I_n(t)/I_{n0} - 1]$ is perfectly uniform and equals $1/\pi$ between $[0 \cdots \pi]$. Using this property, the probability that ac-component $i_n(t) = I_n(t) - I_{n0}$ exceeds a specified value

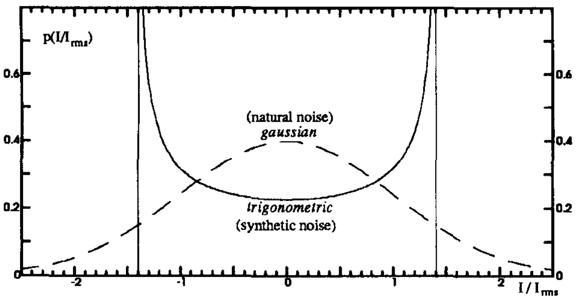

Fig. 3. Probability distribution functions of synthetic noise and (Gaussian distributed) natural noise, normalized to their rms value. The rms value of synthetic noise equals 70% of the maximum ac value ($= 1/\sqrt{2}$).

is

$$\mathbf{P}\{i_n > i\} = 1 - \left(\frac{1}{\pi}\right) \cdot arccos\left(\frac{i}{I_n 0}\right)$$

The probability density is the derivative of this probability and is equal to

$$p_i(i) = \frac{1}{\pi \cdot \sqrt{I_{n0}^2 - i^2}} \qquad |i| \leq I_{n0}. \tag{5}$$

Fig. 3 shows this trigonometric distribution of synthetic noise, in combination with the Gaussian[1] distribution of natural noise. Both curves are normalized to their rms values. The rms value of synthetic noise is equal to

$$(i_{\rm rms})^2 = \int_{-\infty}^{+\infty} p(i) \cdot i^2 \cdot di = \left(\frac{I_{n0}}{\sqrt{2}}\right)^2. \tag{6}$$

E. Measuring Noise Levels Using Non-Gaussian Noise

The major application for synthetic noise generators is the measurement of spectral noise levels. It is performed by detecting the ratio between unknown (natural) noise levels and known (synthetic) noise levels. These measurements require narrow-band filtering, followed by true rms detection of the filtered noise. When two random signals have equal spectral intensity within the resolution band of the filters, but have different probability distributions, the indicated levels are exactly the same. This is the Parseval identity, well-known from Fourier theory. As a result, the non-Gaussian distribution of synthetic noise sets, in *theory,* no restriction to spectral noise measurements.

In practice, however, this difference in distribution is relevant, although of minor importance. It can limit the accuracy of *precision* measurements, such as calibrating portable noise standards. This is a consequence of the way spectral detectors are implemented, such as spectrum analyzers and noise figure meters. Spectral detectors are often equipped with peak-detection instead of true rms detection.

Let us consider two random signals having an equal spectral level but different probability distribution. True rms detection will indicate equal levels, but peak-detection can indicate a (small) difference. We observed up to 0.2 dB difference with an HP8970a noise figure meter, when comparing equal noise levels (according to external true rms detection) with the built-in peak-detector.

For many spectrum analyzers this difference is probably higher because they are additionally equipped with logarithmic IF amplifiers. This is because the nonlinear transfer of these amplifiers change the probability distribution of the (filtered) signals before they reach the peak-detector.

In practice, the uncertainty of noise measurements (for instance on lightwave receivers) is usually much higher than 0.2 dB. This illustrates that the difference in distribution between natural and synthetic noise is often of minor importance.

III. Incoherence Analysis

The two lightwave beams in the composite lightwave signal are to be completely incoherent to generate *random* photocurrents. An adequate incoherent interference requires a "sufficiently" long differential length of the interferometer and a "sufficiently" high random FM modulation level. This section discusses these basic incoherence requirements for "well designed" synthetic noise generators.

A. Definition of Incoherence Threshold

The *incoherence* and *incoherence threshold* are numerical measures that indicate whether a synthetic noise generator is well designed or not. They quantify the random fluctuation in the differential phase $\Delta\alpha$ of the composite signal. Since $\Delta\alpha(t, \tau) = \Delta\varphi(t, \tau) + \Delta\theta(t, \tau) + \omega_0 \cdot \tau$ is the superposition of periodic variations $\Delta\varphi(t, \tau)$, random fluctuations $\Delta\theta(t, \tau)$, and an offset value $(\omega_0 \cdot \tau)$, we define:

$$\text{Incoherence} \stackrel{\text{def}}{=} \Delta\theta_{\rm rms} \tag{7a}$$

$$\text{Incoherence threshold :} \quad \Delta\theta_{\rm rms} \stackrel{\text{def}}{=} \pi. \tag{7b}$$

Do not confuse this (in)coherence with the well known laser quantity *coherence length.* Coherence length is a quantity of the laser alone, and is proportional to $1/\theta_{\rm rms}$, (when all external random modulation is switched off). Our quantity *incoherence* is a property of the total setup, and depends on modulation and interferometer properties as well. The incoherence increases with the differential length (or delay τ) of the interferometer and the depth of random modulation, as discussed in Sections III-B and III-C. Fig. 4 illustrates the relation between the incoherence level and the appearance of the generated photocurrent $I_n(t)$.

Initially, the higher the incoherence, e.g., by deeper random modulation, the more electrical noise is generated. When the incoherence exceeds the *incoherence threshold* value π, then $\cos(\Delta\theta)$ will fluctuate over the full available span. The interference is wrapped in 2π intervals and the current fluctuations span the full available range. The interference between the two lightwave signals will then fluctuate from

[1]Gaussian $p_i(i) = \frac{1}{\sqrt{(2\cdot\pi)}\cdot i_{\rm rms}} \cdot \exp\left(-\frac{1}{2} \cdot (i/i_{\rm rms})^2\right)$.

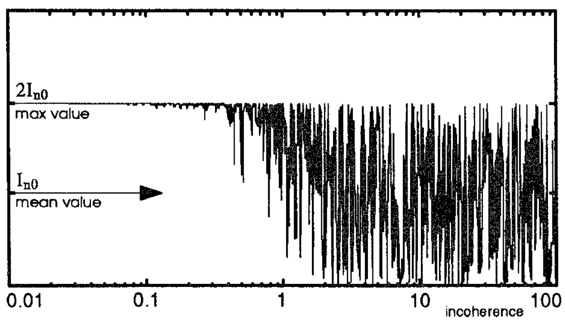

Fig. 4. Simulated oscilloscope view of the synthetic noise current $I_n(t)$, in the case that the random modulation level of $J(t)$ sweeps from zero to a maximum value. Far above the incoherence threshold, the output noise spans the full range, and is suitable for measurement purposes.

perfect extinction to perfect superposition. This makes the total available noise power *independent* of the modulation depth.

Operation far above the incoherence threshold means that the generated synthetic noise is sufficiently random and therefore suitable for measurement purposes.

B. Injection Level for Adequate Incoherence

The incoherence of a synthetic noise generator can be improved by increasing the level of noise injection [3]. This is important when the differential delay time τ of the interferometer is too short, and the coherence time τ_c of the laser (linewidth) is too long. This section derives a relation between the level of $J(t)$ and the resulting incoherence.

The incoherence $\Delta\theta_{\text{rms}}$ can be expressed in the single-sided intensity spectrum S_J of the random modulation signal $J(t)$ using the well-known Parseval identity

$$
\begin{aligned}
(\Delta\theta_{\text{rms}})^2 &= \int_0^{+\infty} \tilde{S}\{\Delta\theta(t,\tau)\}\cdot df \\
(\Delta\theta_{\text{rms}})^2 &= (2\pi\cdot M)^2 \cdot \int_0^{+\infty} \\
&\quad \cdot \tilde{S}\left\{\int_{-\infty}^{t} \{J(t) - J(t+\tau)\}\cdot dt\right\}\cdot df \\
(\Delta\theta_{\text{rms}})^2 &= (2\pi\cdot M)^2 \cdot \int_0^{+\infty} \left|\frac{1}{j\omega}\right|^2 \\
&\quad \cdot \tilde{S}\{J(t) - J(t+\tau)\}\cdot df \\
(\Delta\theta_{\text{rms}})^2 &= (2\pi\cdot M)^2 \cdot \int_0^{+\infty} \left|\frac{1-\exp(j\omega\tau)}{j\omega}\right|^2 \\
&\quad \cdot \tilde{S}\{J(t)\}\cdot df \\
\Delta\theta_{\text{rms}} &= (2\pi\cdot M\cdot\tau) \\
&\quad \cdot \sqrt{\int_0^{+\infty} \text{sinc}^2(f\cdot\tau)\cdot\tilde{S}\{J(t)\}\cdot df}. \quad (8)
\end{aligned}
$$

where $\text{sinc}(x)$is defined as

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}.$$

This incoherence expression holds for injected noise having arbitrary spectra. Two situations are of special interest.

- Injecting with *white noise*, which means that the intensity spectrum of the injected signal is constant over a sufficiently wide frequency interval. This makes $\tilde{S}\{J(t)\} = S_{\text{JO}}$ for all frequencies where $\text{sinc}^2(f\cdot\tau)$ is significant.
- Injecting with *band-limited noise,* which means that the cut-off frequency of the injected signal is lower than $f_c < 1/(2\tau)$. This makes $\text{sinc}^2(f\tau) \approx 1$ for all frequencies in the pass-band.

Under these conditions the expression for incoherence simplifies into one of

$$
\begin{aligned}
[\Delta\theta_{\text{rms}}]_{\text{white}} &\approx (2\pi\cdot M\cdot\tau) \\
&\quad \cdot \sqrt{S_{\text{JO}}\cdot\int_0^{-\infty}\text{sinc}^2(f\tau)\cdot df} \quad (9a) \\
[\Delta\theta_{\text{rms}}]_{\text{band}} &\approx (2\pi\cdot M\cdot\tau) \\
&\quad \cdot \sqrt{1\cdot\int_0^{+\infty} S_J(f)\cdot df} \quad (9b)
\end{aligned}
$$

$$
\begin{aligned}
\text{white noise}: &\quad \Delta\theta_{\text{rms}} \approx 2\pi\cdot M\cdot\sqrt{\tfrac{1}{2}\tau\cdot S_{\text{JO}}} \quad (10a) \\
\text{band-limited}: &\quad \Delta\theta_{\text{rms}} \approx 2\pi\cdot M\cdot\tau\cdot J_{\text{rms}}. \quad (10b)
\end{aligned}
$$

These simplified expressions demonstrate that the incoherence increases with the injection level of random signal $J(t)$ and with the differential delay time τ of the interferometer.

Let us assume a synthetic noise generator injected with band-limited noise, having a laser with an FM sensitivity of $M = 300$ MHz/mA, and a fiber-optic interferometer with $\Delta L \approx 10$ m differential delay length. The cut-off frequency is not exceeding the value $f = 1/(2\cdot\tau) = c/(\Delta L\cdot 2) \approx 10$ MHz. Under these conditions, at least $33\,\mu$A band-limited noise current is required to facilitate operation above the incoherence threshold. On the other hand, when the injected noise is white and equally spread out over 1 GHz, at least $330\,\mu$A noise is required for the same incoherence performance.

C. Incoherence in Absence of Noise Injection

In the absence of external noise injection, all random FM effects have a parasitic origin. Relying on parasitics only requires an interferometer having a very long differential delay length to meet the incoherence requirements. Wang *et al.* [1] reported such a synthetic noise generator using 1 km length, and recommended that the differential delay τ must exceed the coherence time τ_c of the laser.

A rough estimation of the laser linewidth is $\Delta f \approx 2\cdot M\cdot J_{\text{rms}}$ in which $J(t)$ represents all parasitic FM effects. Since the laser coherence time t_c is related to this width by $\tau_c = 1/\Delta f$, Wang's recommendation equals $2\cdot M\cdot\tau_c\cdot J_{\text{rms}} > 1$. This criterion is equivalent to operation above incoherence threshold for injection with band-limited noise, and supports our recommendations.

IV. Frequency Domain Analysis

Operation far above incoherence threshold is a requirement for generating a random photocurrent with maximum rms

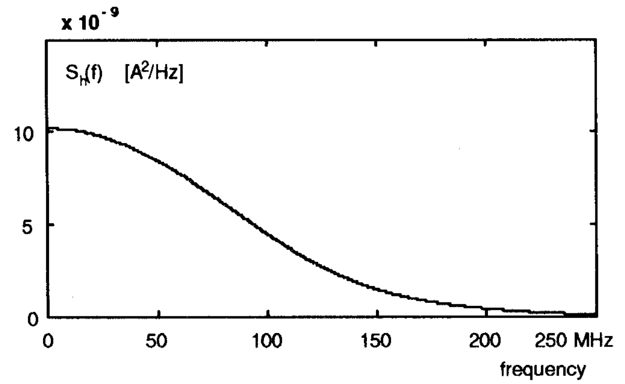

Fig. 5. Simulation of a homodyne spectrum, $S_h(f)$ generated in a setup having $M = 300$ MHz/mA, and $\tau = 0.1\,\mu$s. Band-limited random noise was injected at level $J_{\rm rms} = 1$ mA, using a first order low-pass filter of 100 KHz. The dc photocurrent was normalized to $I_{n0} = 1$ A.

value. It is controlled by the level of $J(t)$. Additional periodical modulation spreads the spectrum of the photocurrent out over nearly twice the FM modulation depth (lightwave). This section derives the synthetic noise spectrum for arbitrary modulation $I(t) + J(t)$, when the level of $J(t)$ is high enough for operating *far* above incoherence threshold.

A. Calculation of Spectral Intensity

The synthetic spectrum that results from periodical FM modulation with $I(t)$ is discrete in nature. It is essentially an equidistant line spectrum (the "frame"). An additional random modulation broadens the individual lines, which results in a comb spectrum. This spectrum will be smoothed into a white noise spectrum when the level of $J(t)$ is sufficiently high [3]. We define the following two spectra: $\overset{\infty}{Q}(n)$ and $S_h(f)$.

- $\overset{\infty}{Q}(n)$ is a discrete Fourier spectrum, representing the magnitude of the individual comb-lines, that originates from periodic modulation with signal $I(t)$ having period $T = 1/f_m = 1/(2\pi\omega_n)$.
- $S_h(f)$ is a single sided intensity spectrum, representing the envelope of the individual comb-lines, that originates from random modulation with $J(t)$.

Let $\varphi_d\{x(t);\, T\}$ represent the discrete Fourier transform of signal $x(t)$ over period T, and let $\tilde{S}\{y(t)\}$ represent the single sided intensity spectrum of signal $y(t)$, then we define

$$\overset{\infty}{Q}(n) \overset{\rm def}{=} \varphi_d\{\exp[j\cdot\Delta\varphi(t,\,\tau)];\, T\} = \text{frame} \tag{11a}$$

$$S_h(f) \overset{\rm def}{=} \tilde{S}\{I_{n0}\cdot\cos[\Delta\theta(t,\,\tau)]\} = \text{homodyne spectrum.} \tag{11b}$$

The differential phase in these expressions is related to the modulation signals as stated in (2a) and (2b). Spectral calculation of random FM signals simplifies significantly when they are modulated far above incoherence threshold. It can be demonstrated [5] that when $\Delta\theta_{\rm rms} \gg \pi$ and when δ_0 represents an arbitrary phase shift, the following properties hold:

$$\tilde{S}\{\sin[\Delta\theta(t)]\} = \tfrac{1}{2}\cdot\tilde{S}\{\exp[(j\cdot\Delta\theta(t)]\} \tag{12a}$$

$$\tilde{S}\{\cos[\Delta\theta(t)]\} = \tfrac{1}{2}\cdot\tilde{S}\{\exp(j\cdot\Delta\theta(t)]\} \tag{12b}$$

$$\tilde{S}\{\cos[\Delta\theta(t)+\delta_0]\} = \tfrac{1}{2}\cdot\tilde{S}\{\exp[j\cdot\Delta\theta(t)]\}. \tag{12c}$$

These expressions illustrate that the intensity spectrum of FM modulated signals is invariant for constant phase shifts when randomly modulated far above incoherence threshold.

Using these incoherence properties, and the well-known property that $\exp[j\cdot\Delta\varphi(t,\,\tau)] = \sum_{n=-\infty}^{+\infty} \overset{\infty}{Q}_n \cdot \exp(j\cdot n\omega_m t)$, we derive for the overall synthetic noise spectrum

$$\begin{aligned}
\tilde{S}\{i_n(t)\} &= \tilde{S}\{I_{n0}\cdot\cos(\Delta\alpha)\}\\
&= \tilde{S}\{I_{n0}\cdot\cos[\omega_0\cdot\tau + \Delta\theta(t,\,\tau) + \Delta\varphi(t,\,\tau)]\}\\
&= \tfrac{1}{2}\cdot(I_{n0})^2\cdot\tilde{S}\{\exp[j\cdot\omega_0\cdot\tau + j\cdot\Delta\theta(t,\,\tau)\\
&\quad + j\cdot\Delta\varphi(t,\,\tau)]\}\\
&= \tfrac{1}{2}\cdot(I_{n0})^2\cdot\tilde{S}\{\exp(j\cdot\omega_0\cdot\tau)\cdot\exp[j\cdot\Delta\theta(t,\,\tau)]\\
&\quad \cdot\exp[j\cdot\Delta\varphi(t,\,\tau)]\}\\
&= \tfrac{1}{2}\cdot(I_{n0})^2\cdot|\exp(j\cdot\omega_0\cdot\tau)|^2\\
&\quad \cdot\tilde{S}\{\exp[j\cdot\Delta\theta(t,\,\tau)]\cdot\exp[j\cdot\Delta\varphi(t,\,\tau)]\}\\
&= \tfrac{1}{2}\cdot(I_{n0})^2\cdot 1\cdot\tilde{S}\{\exp[j\cdot\Delta\theta(t,\,\tau))\cdot\sum \overset{\infty}{Q}_n\\
&\quad \cdot\exp(j\cdot n\omega_m t)\}\\
&= \tfrac{1}{2}\cdot(I_{n0})^2\sum|\overset{\infty}{Q}_n|^2\cdot\tilde{S}\{\exp[j\cdot\Delta\theta(t,\,\tau)]\\
&\quad \cdot\exp(j\cdot n\omega_m t)\}\\
S_i(f) &= \tilde{S}\{i_n(t)\}\\
&= \sum_{n=-\infty}^{+\infty}|\overset{\infty}{Q}_n|^2\cdot S_h(f + n\cdot f_m).
\end{aligned} \tag{13}$$

These expressions demonstrate that the overall synthetic noise spectrum $S_i(f)$ is the superposition of identical spectra $S_h(f)$. Each individual comb-line $|\overset{\infty}{Q}_n|^2\cdot S_h(f + n\cdot f_m)$ equals the homodyne spectrum $S_h(f)$ but is shifted in frequency and scaled by a frame-line magnitude $|\overset{\infty}{Q}_n|^2$.

B. Calculation Example

Let us calculate the output spectrum of a synthetic noise generator, having a differential delay τ optimized for $f_m = 1/\tau = 10$ MHz modulation frequency. This requires a fiber-optic interferometer with approximately 10 m differential length. The FM sensitivity of the laser is assumed to be $M = 300$ MHz/mA.

At first, injecting the setup with 1 $\text{mA}_{\rm rms}$ band-limited noise $J(t)$ results in a homodyne spectrum shown in Fig. 5. The spectral width and shape is highly affected by the injection level, the filter corner frequency, and the filter order. Our example is an arbitrary chosen combination, in which $J(t)$ originates from white noise, filtered by a 100 KHz first order low-pass filter. The associated homodyne spectrum is calculated, using 32 768 Gaussian distributed random numbers (white spectrum), digital low-pass filtering, and standard FFT techniques. The resulting intensity spectrum in Fig. 5

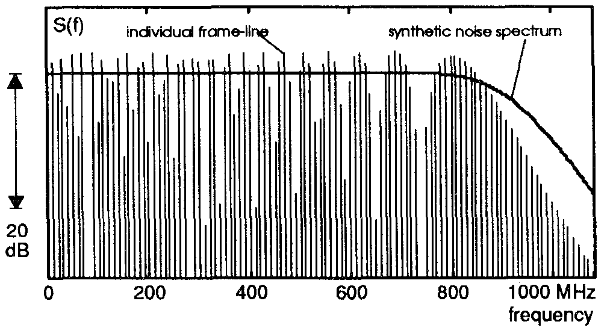

Fig. 6. The total synthetic noise spectrum is the superposition of individual comb-lines, each with envelope $|Q_n|^2\cdot S_h(f+n\cdot f_m)$. When the width of each comb-line spans several lines, the total spectrum smoothes into a (virtually) white noise spectrum. Above 850 MHz, the synthetic noise spectrum exceeds the average level of the (downscaled) frame-lines because of the nonzero linewidth of each comb-line.

is smoothed to simulate the effect of an infinite number of random samples.

Second, an additional modulation signal $I(t)$ spreads the homodyne spectrum out over a wide frequency band. Fig. 6 shows the calculated synthetic noise spectrum for $f_m = 10$ MHz modulation at a level of 3 mA (peak-peak). The total noise bandwidth is proportional to this level, and equals the lightwave frequency sweep (900 MHz). The noise level is proportional to the lightwave power of the laser. We applied triangular modulation, instead of sinusoidal modulation, to perform an equal spreading over the full 900 MHz sweep band. This has been demonstrated in [3].

The deeper the random modulation [the level of $J(t)$], the wider the homodyne spectrum will be, and the lower the spectral ripple in the synthetic noise. In our example, the calculated spectral ripple is less than 0.01 dB below 600 MHz.

The synthetic noise spectrum in Fig. 6 is overlaid with the frame-lines, downscaled to level $2\cdot(I_{n0}\cdot|\overset{\infty}{Q}_n|)^2/f_m$. They illustrate how the envelope of the frame-spectrum is smoothed into the total synthetic noise spectrum by the width of the homodyne spectrum.

C. Power Conservation Identities

Using the well-known Parseval identity for spectra, we obtain for the overall homodyne power and overall frame power

$$\int_0^{+\infty} S_h(f)\cdot df = \langle |I_{n0}\cdot\cos[\Delta\theta(t,\tau)]|^2\rangle_\infty = \frac{1}{2}\cdot(I_{n0})^2 \tag{14a}$$

$$\sum_{n=-\infty}^{+\infty} \overset{\infty}{Q}_n = \langle |\exp[j\cdot\Delta\varphi(t,\tau)]|^2\rangle_\infty = 1. \tag{14b}$$

The first expression equals the square of the rms value $(i_{\rm rms})^2$ of the synthetic noise, which is independent of the modulation depth. Modulation spreads the spectrum out over a wide frequency interval but preserves all available power. The wider the envelope of the frame, the lower the spectral level of the synthetic noise S_i will be.

In a well-designed noise generator, the synthetic noise is equally spread over a wide frequency bandwidth B. This width is user-definable by controlling the magnitude of $I(t)$, which affects the FM modulation depth. Since the power $(i_{\rm rms})^2 \approx S_i\cdot B$ is preserved, the white noise level (far above incoherence threshold) is equal to

$$S_i \approx \frac{(i_{\rm rms})^2}{B} = \left(\frac{I_{n0}}{\sqrt{2B}}\right)^2. \tag{15}$$

D. Total Spectral Noise Level at the Photodiode Output

The total spectral noise level S_{ii} that can be measured at the photodiode output is the sum of the synthetic noise level S_i and the shot noise level S_{i0}. The shot noise equals $S_{i0} = 2\cdot q\cdot I_{n0}$ and is a parasitic effect, associated with the dc photocurrent current I_{n0}. The total output noise is $S_{ii} = S_i + 2\cdot q\cdot I_{n0}$ and decreases when the illumination is reduced by an optical attenuator or when the p-i-n diode is replaced by another type with lower sensitivity.

The shot noise S_{i0} increases proportional to the illumination power, Section IV-B illustrates that the synthetic noise S_i is proportional with (I_{n0}^2), which makes the "noise-current ratio" $(\sqrt{S_i})/I_{n0}$ a characteristic figure of the synthetic noise generator. This ratio μ is preserved under variation of lightwave power and diode sensitivity, and is equal to

$$\mu \overset{\rm def}{=} \frac{\sqrt{S_i}}{I_{n0}} \approx \frac{1}{\sqrt{(2\cdot B)}} = \text{noise-current ratio} \quad \text{(far above the incoherence threshold).} \tag{16}$$

Our definition of the noise current ratio μ is similar to the definition of laser RIN [15], which is commonly specified in dB/Hz using $10\cdot\log_{10}(\mu^2)$:

$$\text{RIN} \overset{\rm def}{=} \frac{S_{ii} - 2\cdot q\cdot I_{n0}}{I_{n0}^2} = \frac{S_i}{I_{n0}^2} = \mu^2. \tag{17}$$

A fair estimation of B in a well-designed synthetic noise generator is twice the modulation depth and equals the maximum differential frequency of the beams in the composite lightwave signal.

Consider a synthetic noise current, having 10 GHz bandwidth and 100 μA dc photocurrent, then its spectral noise level of $\sqrt{S_i} \approx 700$ pA/$\sqrt{\rm Hz}$ exceeds the associated shot noise level of $\sqrt{S_i} = \sqrt{(2q\cdot I_{n0})} \approx 5.7$ pA/$\sqrt{\rm Hz}$ with 42 dB. This illustrates the power efficiency of synthetic noise generators.

V. CONCLUSIONS

In conclusion, we presented a basic theory for lightwave synthetic noise generation. We demonstrated that synthetic noise is non-Gaussian, an aspect that is relevant only for precision measurements (better than 0.5 dB).

We defined the quantity *incoherence* and derived that the incoherence of "well-designed" generator must exceed the level of π. Noise injection improves this incoherence. Simple formulas have been derived to relate the injection level with incoherence in the special case of white or band-limited noise injection.

We derived expressions for synthetic noise spectra, and demonstrated that injection with 1 mA band-limited noise (100 KHz) is adequate to generate white synthetic noise in a setup having 10 m differential delay and 300 MHz/mA FM sensitivity.

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