Extracting the characteristic impedance matrix from multi-port transmission line measurements - DRAFT

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Abstract—We propose two robust algorithms for the extraction of characteristic termination matrices from multi-port measurements, specified in generic S-, Z- or Y-parameters. One algorithm is based on iterative cascade calculations and the other one on eigenvalue calculations. Although the concept of characteristic impedance is well known for achieving reflection-free transmission in homogeneous two-port cables, we have generalized its definition for applying it to arbitrary multi-port devices. Our approach does not restrict itself to homogeneous or symmetric devices, and even reciprocity nor being passive is a requirement. It starts from generic matrix parameters, does not require any information about inductance or capacitance per unit length, and allows for a true black box approach for analyzing and modeling multi-conductor transmission lines like twisted pair telephony cabling.

This paper starts with defining a solid framework of multi-port relations for signal flow and input impedance, shows that well-known two-port relations do not hold anymore in the multi-port case, and demonstrates at the end the algorithms by applying it to an eight-port characterization of a 100m example multi wire twisted pair cable. As a spin-off, this paper has essentially solved the bi-square matrix equation XAX+BX+XC+D=0 on the fly.

Index Terms—Multi-ports, transmission line matrix methods, transmission line theory, characteristic impedance, scattering matrices, multi-conductor twisted pair cables, cable modeling.

1 Introduction

THE need for multi-port analyses on cables arises in all kinds of studies on Gigabit deployments via existing telephony wiring [1], [2], [3]. The present generation DSL modems like G.fast [3] are transmitting wideband signals beyond 100MHz, and are using these cables far beyond the frequency bands they were ever designed for. Under those conditions, the crosstalk between wire pairs can easily exceed the direct transmission through a wire pair [4], [5], [6], [7] and simple two-port approximations will thus fail. Therefore advanced modeling techniques have been developed for twisted pair cabling, initially only for two-port cable modeling [8], [9] and more recently for multi-port analyses [7], [10], [11], [12] as well.

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Multi-port cable modeling requires knowledge about elementary cable properties, such as for instance the characteristic impedance matrix. That impedance is well known from two-port cable analyses to facilitate reflection-free transmission but the concept can be extended to multi-ports. However, these debates have in common that they are restricted to two-ports [13], [14], [15], [16] or rely from a-priory knowledge on cable structure or primary cable parameters, like inductance and capacitance matrices per unit length [17], [18], [19], [20]. And that information is often lacking for real multi-port cables. What is needed is the other way round: a black box approach starting from multi-port (s-parameter) measurements from which relevant cable properties are to be extracted, without any assumption on homogeneity and symmetry.

This paper proposes two robust algorithms for finding the characteristic termination matrices of arbitrary multi-ports, which are measured and specified in S-, Z- or Y-parameters. These algorithms rely on a generalized definition of characteristic termination, fully independent from the concept of reflection-free transmission, and therefore we provide some basic definitions first. Chapter 2 treats waves, voltages and currents equally, discusses how these signals flow through a multi-port, explains how arbitrary multi-port terminations will be observed at the input, and defines the characteristic termination of a multi-port in this manner. Chapter 3 uses that definition to express the problem as solving the so-called characteristic equation. It shows that well-known formulas for two-ports do not apply anymore for multi-ports and in turn elaborates on two different algorithms for extracting the characteristic termination matrices. The first one is based on the cascade calculations proposed in [21] while the second one is based on eigen value calculations. Chapter 4 demonstrates the usability of our algorithms, and applies them to an eight-port measurement on an example multi-wire telephony cable.

2 Multi-port impedance transformation

2.1 Definitions

The signals at the ports of a device can be described as a pair of waves (W_a, W_b) or as a combination of voltages and currents (U,I). It is a matter of preference which of these two are preferred, since both can be interchanged. If we follow all conventions and terminology detailed in [21], then all (incident) waves W_a flow *into* the multi-port, all (reflected) waves W_b flow $away$ from the multi-port, all currents I are directed *into* the multi-port and all voltages $\mathbf{V}$ are from the port terminals (positive) to a common terminal (negative). If the multi-port is linear then these waves are related via the generic matrix expression $\mathbf{W}_b = \mathbf{S} \times \mathbf{W}_a$ and the voltages and currents via $\mathbf{U} = \mathbf{Z} \times \mathbf{I}$ or $\mathbf{I} = \mathbf{Y} \times \mathbf{U}$. The concept of characteristic termination has only a meaning when the multi-port is cascadable, so when half of its ports are labeled as "input" and the other half as "output". Under those conditions the use of matrix decomposition, as formalized in [21], can simplify matters significantly. Therefore our starting point for describing characteristic terminations of multi-ports are the sets of decomposed matrix relations as summarized in table 1. Further details can be found in [21], where this concept has been formalized.

TABLE 1
Five different sets of definitions of the same cascadable multi-port, in terms of decomposed matrix parameters.

$\mathbf{W}_{bi} = \mathbf{S}_{ii} \times \mathbf{W}_{ai} + \mathbf{S}_{io} \times \mathbf{W}_{ao}$	1a
$\mathbf{W}_{bo} = \mathbf{S}_{oi} \times \mathbf{W}_{ai} + \mathbf{S}_{oo} \times \mathbf{W}_{ao}$	1b
$\mathbf{U}_i = \mathbf{Z}_{ii} \times \mathbf{I}_i + \mathbf{Z}_{io} \times \mathbf{I}_o$	2a
$\mathbf{U}_o = \mathbf{Z}_{oi} \times \mathbf{I}_i + \mathbf{Z}_{oo} \times \mathbf{I}_o$	3b
$\mathbf{I}_i = \mathbf{Y}_{ii} \times \mathbf{U}_i + \mathbf{Y}_{io} \times \mathbf{U}_o$	3a
$\mathbf{I}_o = \mathbf{Y}_{oi} \times \mathbf{U}_i + \mathbf{Y}_{oo} \times \mathbf{U}_o$	3b
$\mathbf{W}_{ai} = \mathbf{T}_f \times \mathbf{W}_{bo} + \mathbf{T}_a \times \mathbf{W}_{ao}$	4a
$\mathbf{W}_{bi} = \mathbf{T}_b \times \mathbf{W}_{bo} + \mathbf{T}_r \times \mathbf{W}_{ao}$	4b
$\mathbf{U}_i = \mathbf{A}_v \times \mathbf{U}_o - \mathbf{A}_z \times \mathbf{I}_o$	5a
$\mathbf{I}_i = \mathbf{A}_y \times \mathbf{U}_o - \mathbf{A}_c \times \mathbf{I}_o$	5b

2.2 Multi-ports under arbitrary termination

If a cascadable multi-port is terminated with an arbitrary but known multi-port reflection network $\mathbf{S}_L$ then we will observe a reflection matrix $\mathbf{S}_i$ at the input and a wave transfer matrix $\mathbf{H}_w$ between input with output signals. Something similar applies for voltages and currents, and the full set of definitions to relate signals under terminated conditions is summarized in table 2.

TABLE 2
Three different sets of definitions about input and output signals of the same multi-port under arbitrary termination.

load reflection:	$\mathbf{W}_{ao} = \mathbf{S}_L \cdot \mathbf{W}_{bo}$	1a
input reflection	$\mathbf{W}_{bi} = \mathbf{S}_i \cdot \mathbf{W}_{ai}$	1b
wave transfer	$\mathbf{W}_{bo} = \mathbf{H}_w \cdot \mathbf{W}_{ai}$	1c
load impedance	$\mathbf{U}_o = -\mathbf{Z}_L \cdot \mathbf{I}_o$	2a
input impedance	$\mathbf{U}_i = +\mathbf{Z}_i \cdot \mathbf{I}_i$	2b
voltage transfer	$\mathbf{U}_o = \mathbf{H}_u \cdot \mathbf{U}_i$	2c
load admittance	$\mathbf{I}_o = -\mathbf{Y}_L \cdot \mathbf{U}_o$	3a
input admittance	$\mathbf{I}_i = +\mathbf{Y}_i \cdot \mathbf{U}_i$	3b
current transfer	$\mathbf{I}_o = -\mathbf{H}_i \cdot \mathbf{I}_i$	3c

Both the input reflection and wave transfer matrix can be expressed in the termination reflection $\mathbf{S}_L$. Table 3 summarizes the result of a matrix elaboration on the definitions in table 1 and 2. These equations are expressed by using the Matlab syntax for matrix inversion, where$\mathbf{A}/\mathbf{B} = \mathbf{A} \times inv(\mathbf{B})$ and $\mathbf{A}\backslash\mathbf{B} = inv(\mathbf{A}) \times \mathbf{B}$. Matrix $\mathbb{I}$ refers to the unity matrix.

These equations hold only in forward direction and make no assumptions on reciprocity and/or symmetry of the multi-port. The expressions for the reflection in reverse direction can simply be obtained by swapping matrix $\mathbf{S}_{oi}$ with $\mathbf{S}_{io}$ and $\mathbf{S}_{ii}$ with $\mathbf{S}_{oo}$. A similar approach applies for the expressions with $\mathbf{Z}$ and $\mathbf{Y}$ parameters as well.

TABLE 3
Equations for forward signal transfer and input properties at specified multi-port termination.

via decomposed generic matrix parameters, from $\mathbf{S}_L$, $\mathbf{Z}_L$, $\mathbf{Y}_L$		
waves & reflections	$\mathbf{H}_w = (\mathbb{I} - \mathbf{S}_{oo} \cdot \mathbf{S}_L)\backslash\mathbf{S}_{oi}$	1a
	$\mathbf{S}_i = \mathbf{S}_{ii} + (\mathbf{S}_{io} \cdot \mathbf{S}_L) \times H_w$	1b
voltages & impedances	$\mathbf{Z}_i = \mathbf{Z}_{ii} - \mathbf{Z}_{io}/(\mathbf{Z}_L + \mathbf{Z}_{oo}) \times \mathbf{Z}_{oi}$	2a
	$\mathbf{H}_u = \mathbf{Z}_L/(\mathbf{Z}_L + \mathbf{Z}_{oo}) \times \mathbf{Z}_{oi}/\mathbf{Z}_i$	2b
currents & admittances	$\mathbf{Y}_i = \mathbf{Y}_{ii} - \mathbf{Y}_{io}/(\mathbf{Y}_L + \mathbf{Y}_{oo}) \times \mathbf{Y}_{oi}$	3a
	$\mathbf{H}_i = -\mathbf{Y}_L/(\mathbf{Y}_L + \mathbf{Y}_{oo}) \times \mathbf{Y}_{oi}/\mathbf{Y}_i$	3b
via decomposed chain matrix parameters, from $\mathbf{S}_L$, $\mathbf{Z}_L$, $\mathbf{Y}_L$		
waves & reflections	$\mathbf{S}_i = (\mathbf{T}_b + \mathbf{T}_r \cdot \mathbf{S}_L)/(\mathbf{T}_f + \mathbf{T}_a \cdot \mathbf{S}_L)$	4a
	$\mathbf{H}_w = inv(\mathbf{T}_f + \mathbf{T}_a \cdot \mathbf{S}_L)$	4b
voltages & impedances	$\mathbf{Z}_i = (\mathbf{A}_v \cdot \mathbf{Z}_L + \mathbf{A}_z)/(\mathbf{A}_y \cdot \mathbf{Z}_L + \mathbf{A}_c)$	5a
	$\mathbf{H}_u = \mathbf{Z}_L/(\mathbf{A}_v \cdot \mathbf{Z}_L + \mathbf{A}_z)$	5b
currents & admittances	$\mathbf{Y}_i = (\mathbf{A}_y + \mathbf{A}_c \cdot \mathbf{Y}_L)/(\mathbf{A}_v + \mathbf{A}_z \cdot \mathbf{Y}_L)$	6a
	$\mathbf{H}_i = \mathbf{Y}_L/(\mathbf{A}_y + \mathbf{A}_c \cdot \mathbf{Y}_L)$	6b

The wave transfer matrices ($\mathbf{H}_w$) are not the same as the voltage transfer matrix ($\mathbf{H}_u$) nor the current transfer matrix ($\mathbf{H}_i$) but they are related. Table 4 summarizes these relations, which can be derived from our definitions so far. The same applies for reflection, impedance and admittance, but their relations are not different from the general transformation rules between these three, as summarized in equation (3) of [21].

TABLE 4
Relations between forward signal transfer and input properties at specified multi-port termination.

$\mathbf{H}_w = (\mathbb{I} + \mathbf{S}_L)\backslash\mathbf{H}_u \cdot (\mathbb{I} + \mathbf{S}_i)$	1a
$\mathbf{H}_w = (\mathbb{I} - \mathbf{S}_L)\backslash\mathbf{H}_i \cdot (\mathbb{I} - \mathbf{S}_i)$	1b
$\mathbf{H}_u = \mathbf{Z}_L \cdot \mathbf{H}_i/\mathbf{Z}_i = \mathbf{Y}_L\backslash\mathbf{H}_i \cdot \mathbf{Y}_i$	2a
$\mathbf{H}_u = (\mathbb{I} + \mathbf{S}_L) \cdot \mathbf{H}_w/(\mathbb{I} + \mathbf{S}_i)$	2b
$\mathbf{H}_i = \mathbf{Z}_L\backslash\mathbf{H}_u \cdot \mathbf{Z}_i = \mathbf{Y}_L \cdot \mathbf{H}_u/\mathbf{Y}_i$	3a
$\mathbf{H}_i = (\mathbb{I} - \mathbf{S}_L) \cdot \mathbf{H}_w/(\mathbb{I} - \mathbf{S}_i)$	3b

2.3 Multi-ports under characteristic termination

The previous section is dedicated to arbitrary termination networks, but a special class of terminations can relate the input and output in a remarkable way. This occurs when the reflection matrix $\mathbf{S}_L$ of the termination network is such that the input reflection matrix $\mathbf{S}_i$ becomes equal to it. The impedance of such a termination network is known as image impedance in the domain of (two-port) filter design and as characteristic impedance in the domain of transmission lines, but both concepts are the same and can easily be generalized to arbitrary multi-ports.

In this paper, we will define the characteristic termination of a multi-port as a passive network (if it exists) that causes that the input reflection becomes equal to it, so $\mathbf{S}_i = \mathbf{S}_L = \mathbf{S_c}$, and therefore also equal impedance ($\mathbf{Z}_i = \mathbf{Z}_L = \mathbf{Z}_c$) and admittance ($\mathbf{Y}_i = \mathbf{Y}_L = \mathbf{Y_c}$) at the input. Mark that this generic definition for characteristic termination does not require that the multi-port represents a transmission line, nor to be symmetrical, nor reciprocal, nor passive. When a multi-port is not symmetrical then its characteristic termination matrix in forward direction will be different from its values in reverse direction. So in case $\{\mathbf{S}_c, \mathbf{Z}_c, \mathbf{Y}_c\}$ confuses, we will distinct them by direction as $\{\mathbf{S}_{cf}, \mathbf{Z}_{cf}, \mathbf{Y}_{cf}\}$ and $\{\mathbf{S}_{cr}, \mathbf{Z}_{cr}, \mathbf{Y}_{cr}\}$.

2.4 Characteristic properties of multi-ports

Finding numerical values for $\{\mathbf{S}_c, \mathbf{Z}_c, \mathbf{Y}_c\}$ from multi-port matrix parameters is not trivial, and are therefore discussed in another section 3. But as soon as these values are found, they are associated with a few simple and remarkable properties.

A first property follows directly from the definition. When the multi-port is cascaded with itself, the input reflection matrix of that cascade will again be equal to the load reflection under characteristic termination. And this remains when the cascade grows to infinite length. So in the special case that the multi-port is a homogeneous multi-wire transmission line, and a wave is traveling through that line, that wave will not notice any difference between traveling through an infinite long homogeneous line or a finite line that is loaded with the characteristic termination. Waves in an infinite long homogeneous line will never arrive at the end, so will never be reflected against such an end, and the same absence of reflection will therefore occur if it travels through a finite line that is terminated with its characteristic load. This supports the relevance of finding values for characteristic terminations.

A second property is that the overall transfer matrix $\mathbf{HH}$ of a multi-port self-cascade under characteristic termination is just the matrix product of the individual transfer matrices $\mathbf{H}$. Table 5 summarizes this property for various characteristic transfer matrices. Table 6 summarizes how $\{\mathbf{H}_{cw}, \mathbf{H}_{cu}, \mathbf{H}_{ci}\}$ are interrelated, and these relations are simply a special case of those expressed in 4. In general they are all different, except for the special case that the multi-port has only two-ports. Under those conditions, the decomposition matrices simplify into single scalars and the expressions in table 6 will simplify into$H_{cw} = H_{cu} = H_{ci} = H_c$. But this holds for two-ports only.

A third property is that the generic multi-port parameters of a device can be recovered from its characteristic terminations and associated transfer matrices. Table 7 shows the result of a derivation on how to recover the s-parameters from the set $\{\mathbf{S}_{cf}, \mathbf{S}_{cr}, \mathbf{H}_{cwf}, \mathbf{H}_{cwr}\}$. Similar expressions for $\mathbf{Z}$ and $\mathbf{Y}$ have not been elaborated here but their values can easily be derived via $\mathbf{S} \rightarrow \mathbf{Z}$ or $\mathbf{S} \rightarrow \mathbf{Y}$ transformations as detailed in [21].

TABLE 5
Definition of Transfer and input properties at specified multi-port output terminations.

$\mathbf{HH}_{cw} = \mathbf{H}_{cw} \times \mathbf{H}_{cw} \times \cdots \times \mathbf{H}_{cw}$	1
$\mathbf{HH}_{cu} = \mathbf{H}_{cu} \times \mathbf{H}_{cu} \times \cdots \times \mathbf{H}_{cu}$	2
$\mathbf{HH}_{ci} = \mathbf{H}_{ci} \times \mathbf{H}_{ci} \times \cdots \times \mathbf{H}_{ci}$	3

TABLE 6
Relations between signal transfer and input properties at specified multi-port output terminations.

$\mathbf{H}_{cw} = (\mathbb{I} + \mathbf{S}_c) \backslash \mathbf{H}_{cu} \cdot (\mathbb{I} + \mathbf{S}_c)$	1a
$\mathbf{H}_{cw} = (\mathbb{I} - \mathbf{S}_c) \backslash \mathbf{H}_{ci} \cdot (\mathbb{I} - \mathbf{S}_c)$	1b
$\mathbf{H}_{cu} = \mathbf{Z}_c \cdot \mathbf{H}_{ci} / \mathbf{Z}_c = \mathbf{Y}_c \backslash \mathbf{H}_{ci} \cdot \mathbf{Y}_c$	2a
$\mathbf{H}_{cu} = (\mathbb{I} + \mathbf{S}_c) \cdot \mathbf{H}_{cw} / (\mathbb{I} + \mathbf{S}_c)$	2b
$\mathbf{H}_{ci} = \mathbf{Z}_c \backslash \mathbf{H}_{cu} \cdot \mathbf{Z}_c = \mathbf{Y}_c \cdot \mathbf{H}_{cu} / \mathbf{Y}_{ci}$	3a
$\mathbf{H}_{ci} = (\mathbb{I} - \mathbf{S}_c) \cdot \mathbf{H}_{cw} / (\mathbb{I} - \mathbf{S}_c)$	3b

3 EVALUATING CHARACTERISTIC TERMINATION

3.1 Characteristic equations

Finding numerical values for the characteristic termination of a multi-port is essentially solving an equation that makes the reflection matrix at the input equal to the reflection of its load. These equations are more or less offered via table 3 and equations for $\{\mathbf{S}_c, \mathbf{Z}_c, \mathbf{Y}_c\}$ can be expressed in both generic $\{\mathbf{S}, \mathbf{Z}, \mathbf{Y}\}$ or chain $\{\mathbf{T}, \mathbf{A}\}$ parameters. The equations (1) in table 8 show rephrased versions of the equations (4)-(6) of table 3 and we will call them all characteristic equations. The set of equations (2) in table 8 are essentially the same as those in (1), but only rephrased in matrix format. A simple solution does not exist but we can solve them via an iterative cascade calculation or via eigenvalues calculations. Only in the special case that the multi-port has exactly two ports, and is symmetrical as well, a simple solution does exist. The characteristic impedance is then a scalar and equals to the well-known expression $Z_c = \sqrt{Z_i^{short} \cdot Z_i^{open}}$, where $Z_i^{short} = A_z/A_c = A_z/A_v$ represents the input impedance at shorted output, and where $Z_i^{open} = A_v/A_y = A_c/A_y$ represents the input impedance at open output. Unfortunately this simplicity does not hold for multi-ports.

TABLE 7
Equations to recover s-parameters from characteristic parameters.

$\mathbf{S}_{ii} = (\mathbf{S}_{cf} - \mathbf{H}_{cwr} \cdot \mathbf{S}_{cf} \cdot \mathbf{H}_{cwf})/(\mathbb{I} - \mathbf{S}_{cr} \cdot \mathbf{H}_{cwr} \cdot \mathbf{S}_{cf} \cdot \mathbf{H}_{cwf})$	a
$\mathbf{S}_{oi} = (\mathbf{H}_{cwf} - \mathbf{S}_{cr} \cdot \mathbf{S}_{cf} \cdot \mathbf{H}_{cwf})/(\mathbb{I} - \mathbf{S}_{cr} \cdot \mathbf{H}_{cwr} \cdot \mathbf{S}_{cf} \cdot \mathbf{H}_{cwf})$	b
$\mathbf{S}_{io} = (\mathbf{H}_{cwr} - \mathbf{S}_{cf} \cdot \mathbf{S}_{cr} \cdot \mathbf{H}_{cwr})/(\mathbb{I} - \mathbf{S}_{cf} \cdot \mathbf{H}_{cwf} \cdot \mathbf{S}_{cr} \cdot \mathbf{H}_{cwr})$	c
$\mathbf{S}_{oo} = (\mathbf{S}_{cr} - \mathbf{H}_{cwf} \cdot \mathbf{S}_{cr} \cdot \mathbf{H}_{cwr})/(\mathbb{I} - \mathbf{S}_{cf} \cdot \mathbf{H}_{cwf} \cdot \mathbf{S}_{cr} \cdot \mathbf{H}_{cwr})$	d

TABLE 8
Characteristic equations in forward direction.

$\mathbf{S}_{cf} \cdot \mathbf{T}_a \cdot \mathbf{S}_{cf} + \mathbf{S}_{cf} \cdot \mathbf{T}_f - \mathbf{T}_r \cdot \mathbf{S}_{cf} - \mathbf{T}_b = \mathbb{O}$	1a
$\mathbf{Z}_{cf} \cdot \mathbf{A}_y \cdot \mathbf{Z}_{cf} + \mathbf{Z}_{cf} \cdot \mathbf{A}_c - \mathbf{A}_v \cdot \mathbf{Z}_{cf} - \mathbf{A}_z = \mathbb{O}$	1b
$\mathbf{Y}_{cf} \cdot \mathbf{A}_z \cdot \mathbf{Y}_{cf} + \mathbf{Y}_{cf} \cdot \mathbf{A}_v - \mathbf{A}_c \cdot \mathbf{Y}_{cf} - \mathbf{A}_y = \mathbb{O}$	1c
$\begin{bmatrix} \mathbf{S}_{cf} & -\mathbb{I} \end{bmatrix} \times \begin{bmatrix} \mathbf{T}_f & \mathbf{T}_a \\ \mathbf{T}_b & \mathbf{T}_r \end{bmatrix} \times \begin{bmatrix} \mathbb{I} \\ \mathbf{S}_{cf} \end{bmatrix}$	2a
$\begin{bmatrix} -\mathbb{I} & Z_{cf} \end{bmatrix} \times \begin{bmatrix} \mathbf{A}_v & \mathbf{A}_z \\ \mathbf{A}_y & \mathbf{A}_c \end{bmatrix} \times \begin{bmatrix} \mathbf{Z}_{cf} \\ \mathbb{I} \end{bmatrix} = \mathbb{O}$	2b
$\begin{bmatrix} \mathbf{Y}_{cf} & -\mathbb{I} \end{bmatrix} \times \begin{bmatrix} \mathbf{A}_v & \mathbf{A}_z \\ \mathbf{A}_y & \mathbf{A}_c \end{bmatrix} \times \begin{bmatrix} \mathbb{I} \\ \mathbf{Y}_{cf} \end{bmatrix} = \mathbb{O}$	2c

3.2 Solutions via cascade calculations

A first method for extracting the characteristic reflections from generic multi-port parameters is based on the evaluation of a (near) infinite cascade of equal multi-ports. When that multi-port is lossy (which is common for cables) then the insertion loss of an infinite cascade becomes infinite high and the input reflection matrix $\mathbf{S}_{ii,\infty}$ becomes indifferent from the output termination. So $\mathbf{S}_{ii,\infty}$will then be equal to the characteristic reflection $\mathbf{S}_c$. Our algorithm 1 is essentially a repetitive calculation of self-cascades until the cascade is long enough to make the input reflection sufficiently indifferent from the output termination.

A robust cascade algorithm using decomposed matrix parameters has been proposed in [21] for arbitrary multi-ports. The involved equations for cascading a first multi-port with a second one is summarized in table 9. It relies on decomposed matrix parameters using the sets {$\mathbf{S1}_{ii}$, $\mathbf{S1}_{oi}$, $\mathbf{S1}_{io}$, $\mathbf{S1}_{oo}$} and {$\mathbf{S2}_{ii}$, $\mathbf{S2}_{oi}$, $\mathbf{S2}_{io}$, $\mathbf{S2}_{oo}$} as input, to produce the set {$\mathbf{S}_{ii}$, $\mathbf{S}_{oi}$, $\mathbf{S}_{io}$, $\mathbf{S}_{oo}$} as output. In our case it is sufficient to keep both multi-ports equal, and to concentrate on the input and output reflection matrices $\mathbf{S}_{ii,M}$ and $\mathbf{S}_{oo,M}$ after M cascades. By reusing the results of each previous calculation step we can simply achieve an exponential grow of the number M of cascaded multi-ports. After N calculation cycles we have cascaded in this way the initial multi-port $M = 2^N$ times. This approach works very fast in practice, since an initial insertion loss of only 0.1 dB becomes about 410 dB after just $N = 12$ cycles, so $\mathbf{S}_{ii,4096}$ can easily approximate $\mathbf{S}_{ii,\infty} = \mathbf{S}_c$ in this example within working precision.

The pseudo code in algorithm 1 shows how this can be implemented. The repeat-until loop implements the iteration, and the set of equations in Table 9 are elaborated in each cycle for the case that$\mathbf{S1}_{xx} = \mathbf{S2}_{xx} = \mathbf{S}_{\mathbf{xx}}$. During each iteration step, $\mathbf{S}_{ii,M}$ and $\mathbf{S}_{oo,M}$ grow step-wise towards the characteristics reflection matrices of interest. After each pass the input reflection $\mathbf{S}_{ii}$ is modified into $\mathbf{S}_{ii} + \mathbf{\Delta}_1$, and the output reflection$\mathbf{S}_{oo}$ into $\mathbf{S}_{oo} + \mathbf{\Delta}_2$. The iteration can stop if $M = 2^N$ has become large enough, which occurs when $\mathbf{S}_{ii}$ and $\mathbf{S}_{oo}$do not change anymore within working precision. The code verifies this by checking if the largest coefficients in $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$are small compared to working precision.

In the special case that the multi-port is symmetric, algorithm 1 simplifies into algorithm 2.

TABLE 9
Equations for cascading two multi-ports, $\mathbf{S1}$ and $\mathbf{S2}$, expressed in decomposed scattering parameters

$\mathbf{S}_{QF} = \mathbf{S2}_{oi}/(\mathbb{I} - \mathbf{S1}_{oo} \times \mathbf{S2}_{ii})$	x
$\mathbf{S}_{QR} = \mathbf{S1}_{io}/(\mathbb{I} - \mathbf{S2}_{ii} \times \mathbf{S1}_{oo})$	y
$\mathbf{S}_{ii} = \mathbf{S1}_{ii} + \mathbf{S}_{QR} \times \mathbf{S2}_{ii} \times \mathbf{S1}_{oi}$	a
$\mathbf{S}_{oo} = \mathbf{S2}_{oo} + \mathbf{S}_{QF} \times \mathbf{S1}_{oo} \times \mathbf{S2}_{io}$	b
$\mathbf{S}_{oi} = \mathbf{S}_{QF} \times \mathbf{S1}_{oi}$	c
$\mathbf{S}_{io} = \mathbf{S}_{QR} \times \mathbf{S2}_{io}$	d

Algorithm 1 Evaluation of characteristic reflections of an arbitrary multi-port

```
function [S_cf, S_cr] = CharRefl(S_ii,S_oi, S_io,S_oo)
repeat
    S_QF = S_oi/(I − S_oo · S_ii)
    S_QR = S_io/(I − S_ii · S_oo)
    Δ_1 = S_QR · S_ii · S_oi
    Δ_2 = S_QF · S_oo · S_io
    S_ii = S_ii + Δ_1
    S_oo = S_oo + Δ_2
    S_oi = S_QF · S_oi
    S_io = S_QR · S_io
    Err = max(max(abs(Δ_1) + abs(Δ_2)))
until Err < 10^−14
S_cf = S_ii
S_cr = S_oo
```

3.3 Solutions via eigenvalue calculations

The previous algorithm in section 3.2 is quite convenient in many practical situation, but may not converge when the multi-port is loss-less. So the cascade calculation is suitable for most practical (and passive) transmission lines, but not for the generic multi-port case. Moreover, the cascade method provides only one solution while the characteristic equations in table 8 have multiple solutions. And all these solutions can fulfill the requirement that they make the input reflection equal to the load reflection under characteristic termination. Therefore we developed a second

Algorithm 2 Evaluation of characteristic reflections of a symmetrical multi-port.

function [$\mathbf{S}_c$] = CharRef_Sym($\mathbf{S}_{ii}$,$\mathbf{S}_{oi}$)
% assume $\mathbf{S}_{oo} = \mathbf{S}_{ii}$ and $\mathbf{S}_{io} = \mathbf{S}_{oi}$
repeat
  $\mathbf{S}_Q = \mathbf{S}_{oi}/(\mathbb{I} - \mathbf{S}_{ii} \cdot \mathbf{S}_{ii})$
  $\mathbf{\Delta} = \mathbf{S}_Q \cdot \mathbf{S}_{ii} \cdot \mathbf{S}_{oi}$
  $\mathbf{S}_{ii} = \mathbf{S}_{ii} + \mathbf{\Delta}$
  $\mathbf{S}_{oi} = \mathbf{S}_Q \cdot \mathbf{S}_{oi}$
  $Err = max(max(abs(\mathbf{\Delta})))$
until $Err < 10^{-14}$
$\mathbf{S}_c = \mathbf{S}_{ii}$

algorithm, this one based on eigen values. The associated algorithm 3 is quite simple but the associated explanation not.

In order to solve equation (1a) of table 8, we have to substitute the set of decomposed chain parameters {$\mathbf{T}_f$, $\mathbf{T}_a$,$\mathbf{T}_b$,$\mathbf{T}_r$} by another set of matrices. Therefore we will first decompose the full chain matrix $\mathbf{T}$ into $\mathbf{V} \cdot \mathbf{D}/\mathbf{V} = \mathbf{T}$, where $\mathbf{D}$ represents a square diagonal matrix with all eigenvalues of $\mathbf{T}$ on its diagonal, and where $\mathbf{V}$ is a square matrix with all associated eigenvectors in its columns. Such an eigenvalue decomposition is a standard feature of modern matrix packages, and tools like Matlab offer both matrices with one instruction: $[\mathbf{V}, \mathbf{D}] = eig(\mathbf{T})$. The resulting matrices $\mathbf{V}$ and $\mathbf{D}$ are not unique since any swap of eigenvalues and associated eigen vectors can also fulfill the same decomposition. If, for instance, $\mathbf{qq}$ is a row vector with an arbitrary permutation of all indices between 1 and $2n$, then the expression $\mathbf{V}(:, \mathbf{qq}) \cdot \mathbf{D}(\mathbf{qq}, \mathbf{qq})/\mathbf{V}(:, \mathbf{qq})$ will also be equal to $\mathbf{T}$.

We phrased this property by using the Matlab syntax of matrix manipulation, where $\mathbf{V}(:, \mathbf{qq})$ refers to a matrix with all the columns of $\mathbf{V}$ are swapped according to the indices in $\mathbf{qq}$, and where $\mathbf{D}(\mathbf{qq}, \mathbf{qq})$ refers to a matrix where both rows and columns of $\mathbf{D}$ are swapped according to the indices in $\mathbf{qq}$. Mark that any scaling of eigenvectors (columns in $\mathbf{V}$) will also give valid results, but that behavior is irrelevant for our algorithm.

Once a valid matrix pair of $\mathbf{V}$ and $\mathbf{D}$ has been evaluated via standard eigenvalue calculations, we will decompose each of them further into four sub matrices, one for each quadrant. In other words:

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{uu} & \mathbf{V}_{ud} \\ \mathbf{V}_{du} & \mathbf{V}_{dd} \end{bmatrix} \; and \; \mathbf{D} = \begin{bmatrix} \mathbf{D}_u & \mathbb{O} \\ \mathbb{O} & \mathbf{D}_d \end{bmatrix} \tag{1}$$

The indices in this equation refer to "up" and "dn", and matrix $\mathbb{O}$ refers to a $n \times n$ matrix with all zeros.

By expanding the expression $\mathbf{V} \cdot \mathbf{D}/\mathbf{V}$ with these six sub-matrices, we can express the sub matrices of $\mathbf{T}$ and subsequently of $\mathbf{S}$ in those sub matrices. Table 10 shows the result of that elaboration. By substituting the $\mathbf{T}$ matrices of table 10 into equation (2a) of table 8, we can simplify the characteristic equation into those expressed in equation set (2).

By rephrasing the characteristic equation in this form, it becomes clear that when $(\mathbf{S}_{cf} - \mathbf{V}_{du}/\mathbf{V}_{uu}) = \mathbb{O}$, the entire expression will be nullified, and that the same applies when $(\mathbf{S}_{cf} - \mathbf{V}_{dd}/\mathbf{V}_{ud}) = \mathbb{O}$. So $\mathbf{S}_{cf,1} = \mathbf{V}_{du}/\mathbf{V}_{uu}$ and $\mathbf{S}_{cf,2} = \mathbf{V}_{dd}/\mathbf{V}_{ud}$ are just two possible solutions of the characteristic equation. But any permutation of eigenvalues in $\mathbf{D}$ and associated permutation of eigenvectors in $\mathbf{V}$ can also offer valid solutions in the same manner. So our eigenvalue approach has offered many possible solutions for the characteristic reflection $\mathbf{S}_{cf}$ in forward direction.

The general expression of all those solutions of the characteristic reflection equation is shown in equation (1a) of table 11, where {$\mathbf{D_T}$, $\mathbf{V_T}$} refer to the eigenvalues of the chain matrix $\mathbf{T}$. And since we can follow almost the same approach for $\mathbf{Z}_{cf}$ and $\mathbf{Y}_{cf}$ by performing an eigenvalue decomposition of the chain matrix $\mathbf{A}$ into {$\mathbf{D_A}$, $\mathbf{V_A}$} we can all describe them in the same manner as shown in table 11.

Two questions arise at this point. How many possible solutions do exist and are they all suitable for characteristic termination of multi-port with $2n$ ports?

The first question is simple. At a first glance one may expect the total number of possible permutations of indices. The number of possibilities to fill a vector $\mathbf{q}$ with n different indices between 1and $2n$ equals $\frac{(2n)!}{n!}$, but each permutation of the *same* indices in $\mathbf{q}$ does not change the result of a matrix division. In other words: if $\mathbf{A}$ and $\mathbf{B}$ are arbitrary $n \times n$ matrices and $\mathbf{q}$ an arbitrary permutation of the index vector [1:n] then $\mathbf{A}/\mathbf{B} = \mathbf{A}(:, \mathbf{q})/\mathbf{B}(:, \mathbf{q})$. Therefore the maximum number of different solutions that can be obtained with the equation set (1) in table 11 is much lower. More precisely: "$2n$ over n", or $\binom{2n}{n} = \frac{(2n)!}{(n!)^2}$. This equals 6 for $n = 2$, 20 for $n = 3$, 70 for $n = 4$, etc. It is unclear if more solutions of the characteristic equations will exist, but we assume that this is the maximum. But less is possible in special cases since duplicate solutions are not excluded.

The second question is less simple. An arbitrary multi-port, without restrictions like being reciprocal, symmetrical and/or passive, can indeed have this high number of characteristic terminations. But many of the solutions that we have found with our eigenvalue approach have the property that the aggregate power of reflected waves against such a termination network is higher than the aggregate power of incident waves. This is no problem from a theoretical point of view but unappropriated for practical use. But as soon as the multi-port is passive and lossy, we observed that only one solution represents a passive termination network as well. Therefore we aim for finding a permutation $\mathbf{q}$ that offers us the *principle* solution of the characteristic equation, which is by definition the solution that equals the one we introduced in section 3.2.

The first step to achieve that is to sort the eigenvalues in the diagonal of $\mathbf{D}$ with increasing magnitude (and keep them synchronized with the rows in $\mathbf{V}$) so that $\mathbf{D}_u$ contains the smallest ones and $\mathbf{D}_d$ the largest ones. Then cascade in mind the multi-port M times, resulting in a chain chain matrix of $\mathbf{T}^M = \mathbf{V} \cdot \mathbf{D}^M/\mathbf{V}$. The larger the number M , the better the cascade approximates infinite length and the better its $\mathbf{S}_{ii,M}$ will approximate the characteristic reflection $\mathbf{S}_{ii,\infty} = \mathbf{S}_c$. Since $\mathbf{D}_d$ contains the largest eigenvalues, $\mathbf{D}_u^m$ will soon become neglectable compare to $\mathbf{D}_d^m$ when M increases, and expression (1a) of table 10 for $\mathbf{S}_{ii}$ will simplify into:

TABLE 10
Substitution of decomposed $\mathbf{T}$ and $\mathbf{S}$ parameters by decomposed eigenvalue and eigenvector matrices of $\mathbf{T}$.

$\mathbf{T}_f = (\mathbf{V}_{uu} \cdot \mathbf{D}_u / \mathbf{V}_{du} - \mathbf{V}_{ud} \cdot \mathbf{D}_d / \mathbf{V}_{dd}) \,/\, (\mathbf{V}_{uu}/\mathbf{V}_{du} - \mathbf{V}_{ud}/\mathbf{V}_{dd})$	1a
$\mathbf{T}_r = (\mathbf{V}_{dd} \cdot \mathbf{D}_d / \mathbf{V}_{ud} - \mathbf{V}_{du} \cdot \mathbf{D}_u / \mathbf{V}_{uu}) \,/\, (\mathbf{V}_{dd}/\mathbf{V}_{ud} - \mathbf{V}_{du}/\mathbf{V}_{uu})$	1b
$\mathbf{T}_a = (\mathbf{V}_{ud} \cdot \mathbf{D}_d / \mathbf{V}_{ud} - \mathbf{V}_{uu} \cdot \mathbf{D}_u / \mathbf{V}_{uu}) \,/\, (\mathbf{V}_{dd}/\mathbf{V}_{ud} - \mathbf{V}_{du}/\mathbf{V}_{uu})$	1c
$\mathbf{T}_b = (\mathbf{V}_{du} \cdot \mathbf{D}_u / \mathbf{V}_{du} - \mathbf{V}_{dd} \cdot \mathbf{D}_d / \mathbf{V}_{dd}) \,/\, (\mathbf{V}_{uu}/\mathbf{V}_{du} - \mathbf{V}_{ud}/\mathbf{V}_{dd})$	1d
$\mathbf{S}_{ii} = (\mathbf{V}_{dd} \cdot \mathbf{D}_d / \mathbf{V}_{dd} - \mathbf{V}_{du} \cdot \mathbf{D}_u / \mathbf{V}_{du}) \,/\, (\mathbf{V}_{ud} \cdot \mathbf{D}_d / \mathbf{V}_{dd} - \mathbf{V}_{uu} \cdot \mathbf{D}_u / \mathbf{V}_{du})$	2a
$\mathbf{S}_{oo} = (\mathbf{V}_{uu}/\mathbf{D}_u/\mathbf{V}_{uu} - \mathbf{V}_{ud}/\mathbf{D}_d/\mathbf{V}_{ud}) \,/\, (\mathbf{V}_{\mathbf{du}}/\mathbf{D}_{\mathbf{u}}/\mathbf{V}_{\mathbf{uu}} - \mathbf{V}_{dd}/\mathbf{D}_d/\mathbf{V}_{ud})$	2b
$\mathbf{S}_{oi} = (\mathbf{V}_{ud}/\mathbf{V}_{dd} - \mathbf{V}_{uu}/\mathbf{V}_{du}) \,/\, (\mathbf{V}_{ud} \cdot \mathbf{D}_d / \mathbf{V}_{dd} - \mathbf{V}_{uu} \cdot \mathbf{D}_u / \mathbf{V}_{du})$	2c
$\mathbf{S}_{io} = (\mathbf{V}_{du}/\mathbf{V}_{uu} - \mathbf{V}_{dd}/\mathbf{V}_{ud}) \,/\, (\mathbf{V}_{\mathbf{du}}/\mathbf{D}_{\mathbf{u}}/\mathbf{V}_{\mathbf{uu}} - \mathbf{V}_{dd}/\mathbf{D}_d/\mathbf{V}_{ud})$	2d

$$
\begin{aligned}
&\mathbf{S}_{cf} \cdot \mathbf{T}_a \cdot \mathbf{S}_{cf} + \mathbf{S}_{cf} \cdot \mathbf{T}_f - \mathbf{T}_r \cdot \mathbf{S}_{cf} - \mathbf{T}_b = \\
&= \begin{bmatrix} \mathbf{S}_{cf} & -\mathbb{I} \end{bmatrix} \times \begin{bmatrix} \mathbf{T}_f & \mathbf{T}_a \\ \mathbf{T}_b & \mathbf{T}_r \end{bmatrix} \times \begin{bmatrix} \mathbb{I} \\ \mathbf{S}_{cf} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{S}_{cf} & -\mathbb{I} \end{bmatrix} \times \begin{bmatrix} \mathbf{V}_{uu} & \mathbf{V}_{ud} \\ \mathbf{V}_{du} & \mathbf{V}_{dd} \end{bmatrix} \times \begin{bmatrix} \mathbf{D}_u & \mathbb{O} \\ \mathbb{O} & \mathbf{D}_l \end{bmatrix} / \begin{bmatrix} \mathbf{V}_{uu} & \mathbf{V}_{ud} \\ \mathbf{V}_{du} & \mathbf{V}_{dd} \end{bmatrix} \times \begin{bmatrix} \mathbb{I} \\ \mathbf{S}_{cf} \end{bmatrix} \\
&= +(\mathbf{S}_{cf} - \mathbf{V}_{dd}/\mathbf{V}_{ud}) \times \mathbf{V}_{ud} \cdot \mathbf{D}_d / \mathbf{V}_{ud} \times inv(\mathbf{V}_{dd}/\mathbf{V}_{ud} - \mathbf{V}_{du}/\mathbf{V}_{uu}) \times (\mathbf{S}_{cf} - \mathbf{V}_{du}/\mathbf{V}_{uu}) \\
&\quad -(\mathbf{S}_{cf} - \mathbf{V}_{du}/\mathbf{V}_{uu}) \times \mathbf{V}_{uu} \cdot \mathbf{D}_u / \mathbf{V}_{uu} \times inv(\mathbf{V}_{dd}/\mathbf{V}_{ud} - \mathbf{V}_{du}/\mathbf{V}_{uu}) \times (\mathbf{S}_{cf} - \mathbf{V}_{dd}/\mathbf{V}_{ud}) \\
&= \mathbb{O}
\end{aligned}
\tag{2}
$$

TABLE 11
Characteristic termination matrices expressed in the eigen vector matrices of the chain parameters $\{\mathbf{T}, \mathbf{A}\}$.

$\mathbf{S}_{cf} = \mathbf{V_T}(\mathbf{dn},\mathbf{q})/\mathbf{V_T}(\mathbf{up},\mathbf{q})$	1a
$\mathbf{Z}_{cf} = \mathbf{V_A}(\mathbf{up},\mathbf{q})/\mathbf{V_A}(\mathbf{dn},\mathbf{q})$	1b
$\mathbf{Y}_{cf} = \mathbf{V_A}(\mathbf{dn},\mathbf{q})/\mathbf{V_A}(\mathbf{up},\mathbf{q})$	1c
$[\mathbf{V_T}, \mathbf{D_T}]$ = eig $(\mathbf{T})$ $\Rightarrow \mathbf{V_T} \times \mathbf{D_T}/\mathbf{V_T} = \mathbf{T}$	2a
$[\mathbf{V_A}, \mathbf{D_A}]$ = eig $(\mathbf{A})$ $\Rightarrow \mathbf{V_A} \times \mathbf{D_A}/\mathbf{V_A} = \mathbf{A}$	2b
$\mathbf{up} = [1:n]$	3a
$\mathbf{dn} = \mathbf{up} + n = [1+n:2n]$	3b
$\mathbf{q}$= arbitrary set of n indices between 1 and $2n$	3c

$$
\begin{aligned}
\mathbf{S}_{ii} &\rightarrow (\mathbf{V}_{dd} \cdot \mathbf{D}_d / \mathbf{V}_{dd} - \mathbb{O}) \,/\, (\mathbf{V}_{ud} \cdot \mathbf{D}_d / \mathbf{V}_{dd} - \mathbb{O}) \\
&\rightarrow \mathbf{V}_{dd}/\mathbf{V}_{ud}
\end{aligned}
\tag{3}
$$

So the principle solution of the characteristic equation (2a) of table 8 is essentially $\mathbf{S}_{cf}$= $\mathbf{V}(\mathbf{dn},\mathbf{q})$/ $\mathbf{V}(\mathbf{up},\mathbf{q})$, where index vector $\mathbf{q}$ points to the largest eigenvalues in $\mathbf{D}$. The (Matlab) pseudo code for implementing this all is shown in algorithm 3, and is pretty simple to implement.

Algorithm 3 Evaluation of characteristic reflection via eigen values.

```
function S_cf=CharRefl_eig(T)
n=size(T,1)/2;                  half the matrix size
up=1:n;                         indices up rows
dn= up +n                       indices dn rows
[V,D]=eig(T);                   eigenvalues
[d, qq]=sort(abs(diag(D)));     sorting+indexing
q=qq(dn);                       isolate highest values
S_cf= V(dn,q)/ V(up,q)          characteristic refl.
```

4 EXTRACTIONS FROM MEASUREMENTS

To demonstrate our algorithms in practice, we have measured a twisted pair cable of about 100m, and characterized the wires of two pairs (organized in a quad) as eight-port up to 500 MHz. This cable is a multi wire-pair telephony cable that is commonly used in the Netherlands, and its wires are organized in quads and twisted per quad. More details about the measurements on this cable can be found in [4].

At first we treated four wires in some quad of that cable as an eight-port, and extracted both the forward and reverse characteristic impedance matrices from its (single wire, eight port) s-parameter representation. Figure 1 shows the magnitude of each element of the extracted $\mathbf{Z}_{cf}$ and $\mathbf{Z}_{cr}$ as a function of the frequency. Their values are typical in the order of 50 and 100 ohm.

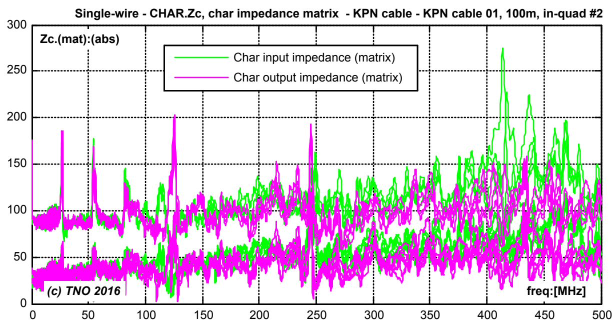

Fig. 1. Extraction of the two characteristic impedance matrices $\mathbf{Z}_{cf}$ and $\mathbf{Z}_{cr}$from four wires measured as an neigh-port.

Secondly we transformed this eight-port of single wires into a four-port of wire *pairs*, to emulate a balanced application where each pair of wires is connected with the outside world via perfect baluns. This is the normal way of how twisted pair cables are being used in practice. Subsequently we extracted the *balanced* characteristic impedance matrix in forward and reverse direction, and the result is shown in figure 2. It may be obvious that we have created another (four-port) device with that and that the characteristic impedance matrix will be different as well. This time, many values in $\mathbf{Z}_{cf}$ and $\mathbf{Z}_{cr}$ are typically in the order of 135 ohm or close to zero.

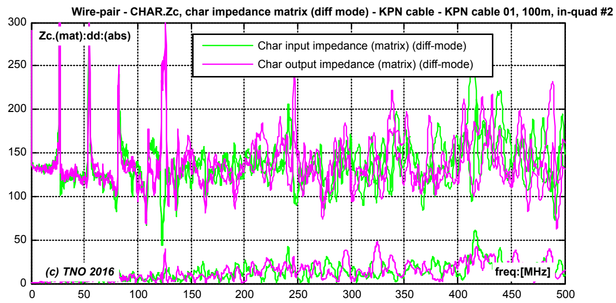

Fig. 2. Extraction of the two characteristic impedance matrices $\mathbf{Z}_{cf}$ and $\mathbf{Z}_{cr}$from two wire *pairs* measured as a four-port. These are the same wires as those used in figure 1.

5 Conclusions

It is well known how to extract the characteristic termination of symmetrical two-ports, but this is not trivial for multi-port. We offered a generalized definition of characteristic termination for multi-ports, not related to traveling waves and not restricted to symmetrical nor reciprocal nor passive multi-ports. The use of waves, voltages or currents are treated equally in this paper, since it is only a matter of preference which one is the most suitable for a certain application.

We demonstrated that the well-known and simple formulas for characteristic impedance of two-ports do not apply any more for multi-ports. We proposed two different algorithms for extracting the characteristic termination from measurements on arbitrary multi-ports. As a spin-off, we derived on the fly the general solution of bi-square matrix equations of the type XAX+BX+XC+D=0. Our algorithms have been applied to many multi-port measurements on twisted-pair telephony cabling and one of these has been provided here to demonstrate our algorithms.

The derivation of the equations in this paper is quite elaborated, but the verification of them is simple. Just use complex random numbers for the s-parameters and verify that all transformations and equations are consistent with each other.

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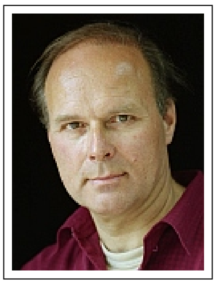

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