Chapter 5
Wideband feedback synthesis

A reliable feedback analysis is a precondition for wideband feedback design. It facilitates assessing transfer functions of feedback amplifier when their configuration is known. On the other hand, feedback design also requires adequate synthesis. This is the reverse problem, dealing with the question what circuit configurations are required to realize transfer functions that meet predefined requirements.

State of the art / manual methods
Nordholt [406] has developed a manual synthesis method for negative feedback amplifiers. This hierarchical design method was initially intended for feedback amplifier design, however it is wider applicable. This synthesis is roughly characterized in the following design steps:

1. *Basic design*, using simplified models for the individual circuit elements. In this design stage, the principle limitations of the overall circuit performance are analyzed, such as bandwidth and noise.
2. *Analysis*, using manual methods or circuit simulation. In this design stage, the analysis of the circuit is complete, using models that take various parasitic effects into account. The parasitic effects that are ignored in the first design step will degrade the overall performance.
3. *Active decoupling*. Suppressing of parasitic effects, using additional sub-circuits. In this design stage, the amplifier stages are supplemented with buffers, cascoding configurations, etcetera, to minimize degradation by parasitic effects. The goal is to approximate the results of the basic design. (stage 1).
4. Sometimes additional sub-sub-circuits are required, to supplement the sub-circuits of the previous design steps.

Essential for this strategy is that the designer is aware of the degradation from optimal feasible performance, at each stage of the design. Furthermore, the performance of the sub-circuits (and sub-sub-circuits) makes lower demands then the performance of the basic design.

State of the art / automated methods
The above synthesis was initially intended as an overall manual method, however, circuit simulation[1] has expanded the applicability. Circuit simulators automate the analysis, but not the synthesis. Attempts have been made to incorporate an automated synthesis. Recent examples of synthesis programs are [501], IDAC [502], BLADES [503],

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[1] Examples of commercially available simulators with powerful N-port facilities are TOUCHSTONE/LIBRA, MDS and Super-Compact/Harmonica. Currently, many others simulators are available with adequate microwave facilities. Further many variants of Berkeley SPICE are commercially available. Touchstone and Libra are courtesy of EEsof inc., MDS of Hewlett Packard and Super-Compact and Harmonica of Compact Software inc.

OASYN [504] OPASYN [505], SEAS [506], OAC [507], ODIN [508], AMPDES [510] and ARIADNE [512].

Due to the complexity of the synthesis problem, the above synthesis programs provide restricted solutions. They may restrict [510] the technology (e.g. CMOS), restrict the number of circuit configurations, use overly simplified models, or rely on adequate reduction of parasitic effects. As a result, they are not applicable to wideband feedback design.

The same applies for the manual use of the above mentioned synthesis theory. The first two design steps are applicable to wideband feedback design. The third step, that must cancel out parasitic effects with additional subcircuits, will eventually fail for increasing bandwidth demands. As a rule of the thumb, above 10% of f_T for second order loops and above 2% of f_T for third order loops, parasitic effects becomes crucial. The sub-circuits will then add additional parasitic effects that may degrade the overall performance instead of improving it.

Wideband feedback design is characterized by parasitic effects that are dominant. So far, there is in general no answer to the question of dealing with parasitic effects that modifies the dominant behavior of the feedback loop.

A novel approach

This chapter provides novel mathematical tools, to synthesize the stability compensation of the loop. It relies on the generalized feedback analysis of the previous chapter 4. that takes fully accounts for parasitic effects.

The proposed synthesis method is partly manual and partly automated. A (tabular) circuit simulator, with augmented facilities, is required to extract the loop gain using the theory of chapter 4. The required transfer functions of the compensation networks in the loop are synthesized with a novel compensation algorithm, that is discussed in this chapter 5. Additional (conventional) synthesis remains required to translate the synthesized compensation transfer into practical circuits.

The highlights of this chapter, which are developed in this study, are:

- Development of a generalized theory on passive compensation networks, that is suitable for automated compensation synthesis.
- Development of an automated synthesis algorithm that predicts the required compensation, in terms of poles and zeros. It has found solutions that are commonly excluded from being feasible (see example in section 5.3.4).

5.1. Aperture analysis of feedback amplifiers

When the loop $\mathring{H}(s) = A(s)\cdot\beta(s)$ is closed, the system gain is controlled by feedback. In general, the loop requires special compensation measures to avoid a feedback system that is unstable or that has resonant peaks in its transfer function.
Compensation syntheses requires an adequate definition for the design goal, and an adequate estimation of the design limitation. The ratio between desired system gain and realized system gain is an intuitive measure to study the pass band limitations of the system. This text refers to this pass band transfer function with the new term *aperture.*
Defining the design goal is equivalent with specifying the aperture. The desired (generic) passband is selected from a class of low-pass filters, selected by the designer. This normalized transfer function is subsequently scaled to its actual value, using the available bandwidth as scaling factor.
This section analyses the aperture in terms of available bandwidth and suitable normalization of the bandpass transfer function.

At first, the concept *aperture* is defined. Some textbooks use the term *closed loop gain* for what will be defined here as "virtual-aperture" while other textbooks use this term for what is defined here as "realized system gain". To avoid further confusion, this study has made up the new term *aperture*.
This study has developed a formal description for the design goal of optimal feedback loops. This formal approach has forced to make a clear distinction between the commonly used *virtual-aperture* and the here proposed *effective-aperture*.

Further, this study has resulted in reliable methods to calculate the available bandwidth. These methods are robust and preserve their validation for feedback loops with parasitic poles and zeros. Finally, this study has resulted in a unified method to specify the generic transfer that is realizable for the passband.

5.1.1. Definition of virtual-aperture and effective-aperture

In general, a loop without any stability compensation will oscillate or will have resonant peaks. The aim of compensating networks is the improvement of the system gain. Since the desired system gain is not necessary a constant, as in case of an integrator or an equalizer, the direct use of system gain is not a convenient design goal. A general applicable design goal is the ratio between what was intended and what is realized.
This text uses the name *aperture* for transfer functions that describe the ratio between the system gain that is desired and the system gain that is realized. In figurative language: the aperture separates the desired gain from the realized gain by a low-pass filter that limits the signal bandwidth when it passes this virtual 'opening'. The design goal is an aperture value close to one, over the widest possible frequency band.
A formal analysis requires an unambiguous definition of desired system gain; a definition that is invariant for any change in compensation. When $(\beta, \varepsilon, \nu)$ refer to the parameters of the compensated feedback network and $(\beta_0, \varepsilon_0, \nu_0)$ to the same parameters of the uncompensated network, then the desired system gain is defined as follows.

forward leakage:	$A_{t0} = \rho$
realized system gain:	$A_t = -(\varepsilon{\cdot}\nu/\beta){\cdot}(A\beta)/(A\beta-1) + \rho$
asymptotic system gain:	$A_{t\infty} = -(\varepsilon{\cdot}\nu/\beta) + \rho$
virtual system gain:	$A_{tv} = -(\varepsilon{\cdot}\nu/\beta)$
desired system gain:	$A_{tt} = -(\varepsilon_0{\cdot}\nu_0/\beta_0)$

Our definition of desired system gain is valid when the forward leakage (ρ) is small, compared to the system gain. Furthermore, we assume that an unambiguous distinction can be made between compensated and uncompensated state. This distinction is not always evident and has led us to introduce in this text two aperture definitions.
We define *effective-aperture* that is of general applicability, and *virtual-aperture* that is of limited applicability. They are defined as follows:

$$\text{Effective-aperture} = \frac{\text{RealizedGain} - \text{Leakage}}{\text{DesiredGain}} = \frac{\mathring{H}}{\mathring{H}-1}\cdot\frac{\varepsilon{\cdot}\nu{\cdot}\beta_0}{\varepsilon_0{\cdot}\nu_0{\cdot}\beta}$$

$$\text{Virtual-aperture} = \frac{\text{RealizedGain} - \text{Leakage}}{\text{VirtualGain}} = \frac{\mathring{H}}{\mathring{H}-1}$$

$\mathring{H}(s) = A(s){\cdot}\beta(s)$

Compensation is used to realize a system gain that approximates the *desired* system gain. The effective-aperture is a handy figure of merit for studying the compensation. An adequate compensation forces the effective-aperture value to one within the frequency band of interest, without resonating peaks and oscillations.
When all compensation is concentrated in the forward amplifier, the virtual-aperture is equal to the effective-aperture. A distinction between those two apertures is then irrelevant. In all other cases the virtual-aperture is a misleading measure for assessing the compensation. Nevertheless, it is a commonly used measure, since the loop gain is the only transfer function that is required for its evaluation.

In order to demonstrate the use of effective-aperture and virtual-aperture in practice, it is convenient to model the compensation as though it is concentrated in two distinct networks. Since the compensation is most effective in A(s) and β(s), this example assumes that the compensation does not affect ε(s) and ν(s). The loop gain is then split-up in the following basic elements:

uncompensated forward gain	$A_0(s)$	
uncompensated feedback factor	$\beta_0(s)$	
profiled compensation	$H_{cs}(s)$	*cascaded with* $A_0(s)$
phantom compensation	$H_{cp}(s)$	*cascaded with* $\beta_0(s)$

The application of the words *profiled* and *phantom* is clarified in a succeeding section 5.1.2. In this section these names are merely used to distinguish the compensation transfer in the forward amplifier (profiled) and the compensation transfer in the feedback network (phantom).

Figure 5.1 shows two sets of equivalent flow diagrams that represent the system feedback in the case (5.1a) that the compensation in the loop is made explicitly visible and in the case (5.1b) that the compensation is embedded. In the first example the

desired system gain and the associated effective-aperture are in evidence. In the second example the virtual gain and the virtual-aperture are used, since no information is available on the phantom compensation.
This figure illustrates that the main signal path is split into two parts: a gain block with ideal characteristics and a (low-pass) filter block that represents all bandpass limitations. It demonstrates that the effective-aperture meets the requirements of the design goal, for which the bandwidth must be maximized and the passband equalized. In this example, the effective-aperture reduces to:

$$\text{Effective-aperture} = \frac{\mathring{H} / H_{cp}}{\mathring{H} - 1} = \frac{1/H_{cp}}{1 - 1/\mathring{H}}$$

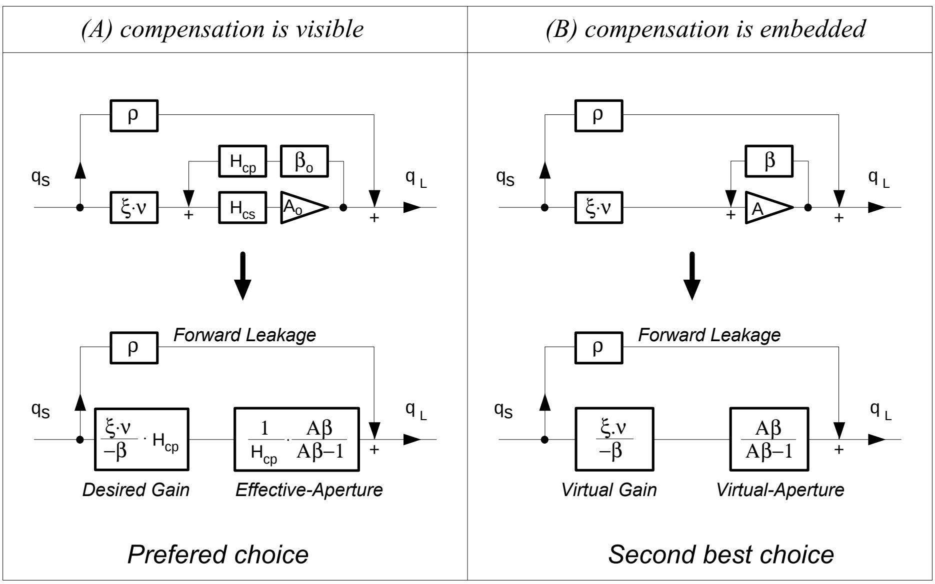

Fig 5.11 *Distinction between effective-aperture and virtual-aperture. The aperture is the transfer of a virtual low pass filter that limits the bandwidth of the gained signal. The effective-aperture is a measure that remains valid when the compensation is (partly) phantom ($H_{cp} \neq 1$). The virtual-aperture is often used, however will be misleading for bandpass analysis. In other situations, virtual-aperture and effective-aperture are equal.*

5.1.2. Bandwidth analysis

The estimation of the available bandwidth of the effective-aperture has been a standard design aspect from the very beginning of wideband amplifier design. It serves as an initial design goal and may be used to distinguish efficient from inefficient compensation methods. This has resulted in the well-known gain-bandwidth product (*GB-product*) for first order feedback loops and, more recently [406], in the loop gain-poles product (*LP-product*) for all-pole higher order feedback loops.
In its current form, the LP-product is not suitable to predict the available bandwidth for loops with annoying parasitic poles and zeros. This is because the LP-product is restricted to *dominant* singularities, that cannot be extracted when the available bandwidth is unknown. As a result, a complementary algorithm is required.
This study has resulted in a robust algorithm that predicts the available bandwidth, irrespective of the presence of parasitic poles and zeros.
For reasons of completeness, the intuitively based design process commonly followed are reviewed here. The aim of this subsection is, however, to develop a method for formal analysis as required by automated synthesis.

Definitions
To begin, the concept *available* is introduced in the context of asymptotic approximations of the effective-aperture. In a well-designed feedback amplifier, the effective-aperture is close to one, over the widest possible frequency band.
The compensation affects this bandwidth in such a way that passive compensation degrades the bandwidth while active compensation improves it. This is because active compensation networks are amplifiers that provide additional loop gain, and an overall increase in loop gain may result in increased bandwidth. Passive compensators, on the other hand, are attenuators that reduce the loop gain and possibly the bandwidth.
This chapter is restricted to passive compensation. These compensation networks are optimal when they preserve all available high-frequency gain. This facilitates the realization of amplifiers with the widest bandwidth.

In a well-designed feedback amplifier, the effective-aperture is constant over a wide frequency band, tending to zero at very high frequencies. Since stability problems are mainly reserved for mid-band frequencies, the compensation must restrict itself to midband frequencies, and must leave the effective-aperture unchanged for zero and infinite frequencies. This facilitates a simplified calculation of the asymptotic effective-aperture without further knowledge of the compensation.

Phantom- and profiled compensation influence the effective-aperture in a different way because of their different location in the loop. Various loop gain aspects can therefore be isolated to simplify the asymptotic evaluation for the effective-aperture:

profiled compensation:	$H_{cs}(s)$	$g_s = H_{cs}(0)$
phantom compensation:	$H_{cp}(s)$	$g_p = H_{cp}(0)$
uncompensated loop gain:	$H_0(s) = A_0(s)\cdot\beta_0(s)$	$g_0 = H_0(0)$
profiled loop gain:	$H_s(s) = A_0(s)\cdot\beta_0(s)\cdot H_{cs}(s)$	$g_s = H_s 0)$
total loop gain:	$\mathring{H}(s) = A_0(s)\cdot\beta_0(s)\cdot H_{cs}(s)\cdot H_{cp}(s)$	$g = \mathring{H}(0)$

Figure 5.2 illustrates the difference between profiled loop gain and total loop gain for an example of a compensated amplifier with three dominant poles. Two (phantom) zeros in the feedback network are added to the loop, to provide the overall compensation. They

modify the loop gain at high frequencies, which causes the difference between profiled and total loop gain.

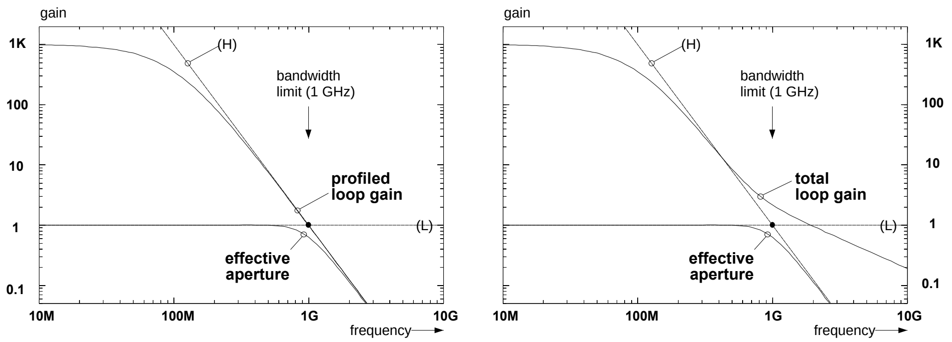

Fig 5.2 *Difference between profiled loop gain and total loop gain for a third order loop with two phantom zeros. The point of intersection of the two asymptotic lines (H) and (L) is the bandwidth limit (1 GHz) of the effective-aperture.*
The profiled loop gain predicts the asymptotic line (H), and is a good measure for the available bandwidth. This does not hold for the total loop gain.

Asymptotic approximations

In a well-designed compensated amplifier, the dominant order of the profiled part of the loop gain is higher than the dominant order of the phantom compensation. This property facilitates an asymptotic approximation of the effective-aperture for $\omega \rightarrow \infty$. Neglecting the high frequency transfer of the phantom compensation, with respect to the profiled part of the loop gain, results in:

$$A_{sa}(s) = \frac{1/H_{cp}(s)}{1 - 1/\mathring{H}(s)} = \frac{1}{H_{cp}(s) - H_{cp}/\mathring{H}(s)(s)} = \frac{1}{H_{cp}(s) - 1/H_s(s)}$$

$$\Rightarrow \left\{ \begin{array}{llll} A_{sa}(s) & \rightarrow & -H_s(s) & (\text{for } \omega \rightarrow \infty) \\ A_{sa}(s) & \rightarrow & \dfrac{1}{H_{cp}(0)} = \dfrac{1}{g_p} & (\text{for } \omega \rightarrow 0) \\ |A_{sa}(j\omega)| & \approx & \dfrac{1}{|g_p| + 1/|H_s(j\omega)|} & \end{array} \right\}$$

The above approximations uses the property that the profiled loop gain is significantly higher than one for low frequencies and tends to zero for high frequencies. As a result, the transfer of the effective-aperture tends to $(1/g_p)$ in the pass band, and tends to $-H_s(s)$ for higher frequencies.
Figure 5.2 demonstrates the impact of this approximation. The close match between effective-aperture and its asymptotic approximation facilitates a first estimation of the

available bandwidth on the basis of asymptotic lines. Note that the high frequency asymptotic line is not an approximation of the *total* loop gain but of a stripped variant, the *profiled loop gain*.

The generalized bandwidth algorithm, in two steps

Secondly, a two-step procedure is described for determining the available bandwidth, that uses the asymptotic properties of the effective-aperture. The first step is a rough estimation, based on the asymptotic corner frequency of the profiled loop gain magnitude. The second step is a refinement step that uses this estimation to deflate the loop gain to its dominant form, using the reduction techniques of section 4.5. The asymptotic corner frequency of the dominant form is then determined, this being a generalization of the LP-product.

Manual synthesis can not use the improved accuracy of the second refinement step due to the approximate nature of manual methods. When manual synthesis methods are adequate, the two-step approach does not result in significantly improved results.

We have observed that the here proposed refinement step is crucial for automated synthesis in those cases in which manual methods fail due to the complexity of the feedback loop. This is because the available bandwidth estimation is an important input parameter for the automated synthesis algorithm that is described in section 5.3. The refinement step has enabled the algorithm to predict realistic solutions where it would otherwise have failed. The generalized definition of the dominant loop gain, see section 4.5.3, is essential to the success of this refinement step.

Step 1*: Global asymptotic corner frequency.* A rough estimation of the available bandwidth is the global asymptotic corner frequency. This frequency ω_b is defined as the intersection point ω_b between low frequency approximation $|A_{sa}(0)| \approx 1/|g_p|$ and the high frequency approximation $|A_{sa}(j\omega)| \approx H_s(j\omega)$ of the effective-aperture. Its value is close to the frequency where the product of phantom dc-gain and profiled loop gain has the value one: $|g_p \cdot H_s(j\omega_b)| = 1$.

For manual analysis it is satisfactory to draw a Bode plot of the profiled loop gain and to read off the intersection point with the unity gain line. Automated analysis may follow the same procedure, using a standard iterative root finding method.

From the standpoint of software implementation, it is convenient to re-use the available polynomial root finding algorithm for this purpose. The corner frequency ω_b is then evaluated as below by the calculation of polynomial roots.

$$|g_p \cdot H_s(j\omega_b)| = |g_p \cdot T_s(j\omega_b)/N_s(j\omega_b)| = 1 \qquad \textit{is a rational function}$$

$$|g_p \cdot H_s(j\omega_b)|^2 = |g_p|^2 \cdot H_s(+j\omega_b) \cdot H_s(-j\omega_b) = 1$$

$$|g_p|^2 \cdot T_s(+j\omega_b) \cdot T_s(-j\omega_b) - N_s(+j\omega_b) \cdot N_s(-j\omega_b) = 0$$

$$\boxed{\{j\omega_b\} \approx \textit{roots}\Big(|g_p|^2 \cdot T_s(+j\omega_b) \cdot T_s(-j\omega_b) - N_s(+j\omega_b) \cdot N_s(-j\omega_b)\Big)}$$

In general, a set of real, purely imaginary and complex pairs will be found as solution for $\{j\omega_b\}$. The purely (positive) imaginary roots are of interest, and usually there is a unique solution. In the exceptional case that there exist more than one intersection between profiled loop gain and the unity gain line, the highest intersection is will be the desired corner frequency.

Step 2: *Dominant asymptotic corner frequency.*
The dominant asymptotic corner frequency ω_0 is an improved estimate of the available bandwidth is . The definition of ω_0 is similar to that of the global asymptotic corner frequencies ω_b differing in the use of a dominant approximation of the profiled loop gain. It requires the preceding calculation of the global value ω_b to supply the corner frequency for the dominant deflation algorithm of section 4.5.3.
From the magnitude approximation of the effective-aperture we obtain:

$$|A_{sa}(j\omega)| \approx \frac{1}{|g_p| + 1/|H_s(j\omega)|} \approx \frac{|A_{sa}(0)|}{1 + |\omega/\omega_0|^{n-m}}$$

$$H_s(s) \approx g_s \cdot \frac{\prod_{r=1}^{r=m}(1-s/z_r)}{\prod_{k=1}^{k=n}(1-s/p_k)} \qquad \textit{(dominant approximation)}$$

$$|A_{sa}(0)| = \frac{1}{|g_p| + 1/|g_s|} = \frac{|g_s|}{|g_p \cdot g_s| + 1} = \frac{|g_s|}{|g_0| + 1}$$

$$|A_{sa}(j\omega)| \approx |H_s(j\omega)| \approx |g_s| \cdot \prod|p_k| / \prod|z_r| \cdot |1/j\omega|^{n-m}$$

$$\boxed{\omega_o = \sqrt[\eta]{(1+|g_0|) \cdot (\prod|p_k|)/(\prod|z_r|)} \qquad \eta=(n-m)}$$

$\{p_k\}$ = *all dominant profiled poles*
$\{z_r\}$ = *all dominant profiled zeros*
$g_0 = g_s \cdot g_p$ = *total DC loop gain*
n, m = *number of dominant profiled loop gain poles and zeros*

The total DC loop gain and the dominant poles and zeros of the profiled loop gain are the only parameters that determine the dominant asymptotic corner frequency.

5.1.3. Bandpass analysis

The available bandwidth of the effective-aperture is in essence a frequency scaling factor of all possible generic low-pass responses for the passband. The bandpass transfer function comprises the whole effective-aperture and must be flattened and made free of resonance.
Synthesis algorithms for compensation require a precise definition of the desired bandpass transfer function, such as maximally-flat or equal ripple amplitude, or linear-phase. This section describes some standard transfer functions, adopted and modified from low-pass filter theory, that match the available bandwidth definition of section 5.1.2.

Figure 5.3 shows some examples of standard low-pass all-pole transfer functions for various transfer orders. These plots are normalized with respect to their asymptotic corner frequency, which is defined as the point of intersection between the two asymptotic Bode lines for low and high frequencies.
The asymptotic corner frequencies proposed here differ from the -3 dB corner frequency that is commonly used in filter theory. This is to link better up with the available bandwidth theory of section 5.1.2. The exact definitions of these modified curves and the algorithms for the associated transfer coefficients are developed during this study, and summarized in appendix *I*.

Compensation synthesis starts from the calculation of the available bandwidth and from the selection of an appropriate bandpass curve. The choice of the low-pass curve is an important design parameter, and is application dependent. On account for what grounds must these choices be based?
One of the most important criterion for digital signals is the minimization of overshoot in the step response. For a set of modulated signals, such as TV channels, a magnitude response with maximum bandwidth (–3 dB) might be relevant. With respect to the three standard transfer functions, relevant properties for a given asymptotic corner frequency are summarized below:

- When the low-pass transfer is close to a Bessel transfer function, the group delay characteristic is maximally flat and the step response has no overshoot.
- When the transfer is close to a Butterworth transfer function, the magnitude response is maximally flat and has a wider bandwidth. Note for digital signals that the lack of ripple in the amplitude response is not equivalent to a lack of overshoot in the step response. As a result, the rise and fall times of the Butterworth step response are shorter than for the Bessel response. This benefit is accompanied by some overshoot in the step response (e.g. 10%).
- The bandwidth of a Chebyshev transfer function, with equal asymptotic corner frequency, is even wider. This is, however, at the cost of significantly higher overshoot.

It is quite risky to restrict the passband analysis to frequency domain analysis alone. Using a frequency domain circuit simulator as a design tool, can easily lead to amplifiers with a seemingly excellent magnitude response, that nevertheless exhibits oscillatory behavior. Although the required information is available in a Bode plot, the difference in phase response between a parasitic pole slightly to the left or slightly to the right of the imaginary axis is hardly noticeable. It is therefore necessary to observe the pole-zero patterns of the aperture of the feedback loop, e.g. as a root-locus plot of the loop gain. The pole-zero patterns in figure 5.3 of characteristic transfer functions are useful for pattern recognition.

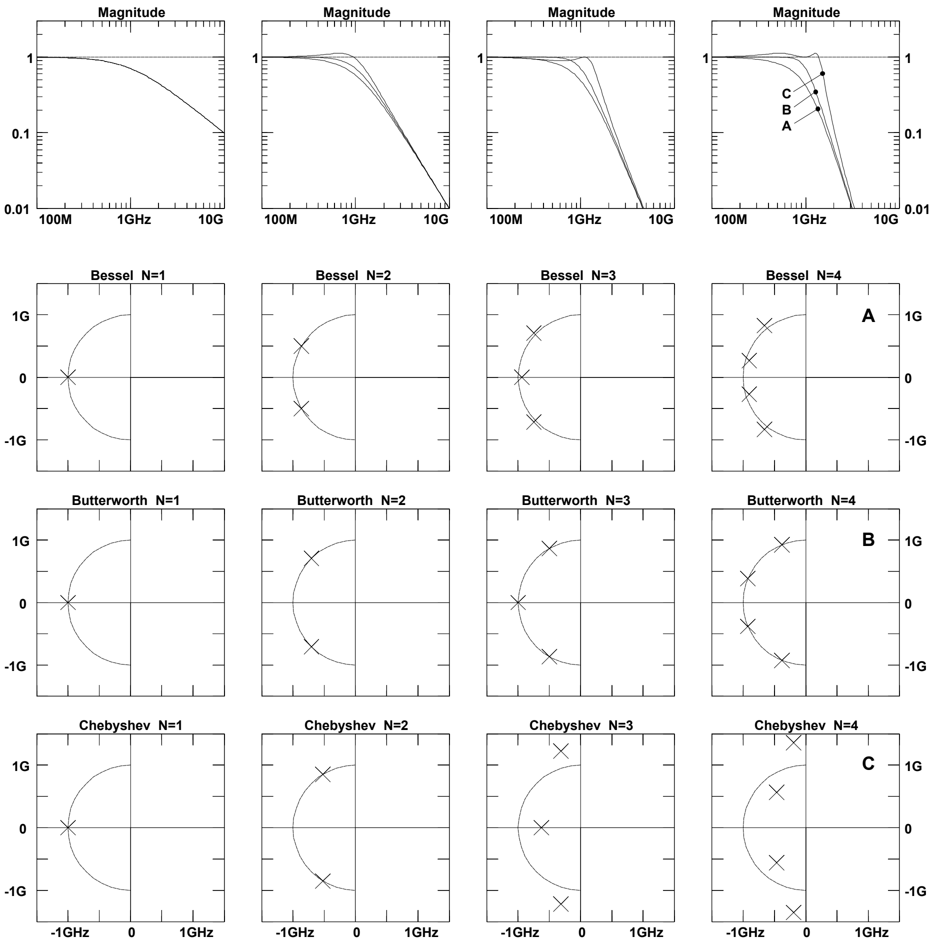

Fig 5.11 *Pole-zero patterns and magnitude plots of various standard low-pass filters: Bessel (A), Butterworth (B) and a 1 dB Chebyshev (C) filter.*

5.1.4. Conclusions

In conclusion, we defined a transfer to enable a designer to define an adequate design goal for feedback synthesis purposes. This so called *aperture* represents the ratio between desired system gain and realized system gain, and has a low-pass transfer function.
Two different apertures were defined: the *effective-aperture* that is of general applicability and the *virtual-aperture* that is of limited applicability. The use of the effective-aperture is preferred, however it requires the transfer function of the phantom compensator in the loop gain. When this is impossible, the designer must manage a restricted synthesis with the virtual-aperture.

We developed a robust algorithm that estimates the available bandwidth from the asymptotic behavior of the effective-aperture. Since a rough estimation may fail when designing complex feedback loops, an additional refinement step was developed. When future circuit simulators give full support to bandwidth prediction, then the interaction between simulator and designer will be as follows:

- The designer splits the circuit in a forward amplifier and a feedback network.
- The simulator extracts the superposition parameters and all the poles and zeros of the loop.
- The designer indicates what feedback singularities are inserted for phantom compensation.
- The simulator provides the available bandwidth.

When the simulator gives full support to the synthesis of section 5.3, then it also provides the required transfer functions for an (additional) phantom compensator and an (additional) profiled compensator.

The available bandwidth is in essence a frequency scaling factor of all possible generic low-pass responses for the passband of the effective-aperture. Various all-pole low-pass responses are analyzed to enable the designer in defining unambiguous design goals for synthesis purposes. Various selection criteria are discussed for three generic low-pass responses: Chebyshev, Butterworth and Bessel. The concept of asymptotic corner frequency is described for low-pass transfer functions to facilitate an adequate normalization .

5.2. Compensation techniques for feedback amplifiers

Compensation is aimed at modifying the loop gain so that the effective-aperture better approximates a predefined transfer function. Practical compensation is confined to physically realizable networks. A number of successful compensation techniques have been described in the literature, by Ghausi and Pederson [404], Cherry and Hooper [405] and Nordholt [406]. These techniques are analyzed primarily from an implementation point of view.

Benefit of optimizers
Automated optimization techniques are very effective compensation tools when the circuit configuration is chosen. The optimizer adjusts various element values simultaneously in an iterative way, to fulfill a predefined design goal. The commercial available simulators Touchstone© [124], Libra© [125], and MDS© [126] give full support to this approach. They allow a hierarchy of circuit blocks and permit the use of variable labels to define relations between tunable elements.
The aperture analysis of section 5.1 provides reliable predictions of the aperture (available bandwidth and generic passband) and is of valuable use to define that design goal. The more realistic this goal is, the more effective an optimizer will be.
The classic paper by Temes and Calahan in 1967 [411] was one of the earliest to formally advocate the use of iterative optimization in circuit design. Bandler and Chen [412] described in 1988 a detailed overview of various methods and summarized more than 100 references to relevant papers. Most optimizers are implemented for tabular circuit simulators. An implementation for an analytical circuit simulator is mentioned in [509,510].

Restrictions of optimizers
Automated optimization techniques require circuit configurations and tuning elements as input. When a solution exists, when adequate tuning elements with proper starting value are chosen and when parasitic effects are adequately minimized by additional sub-circuits (buffering, cascoding), then optimizers may be very effective.
On the other hand, they will never find a solution when the compensating elements are inadequate. Compensation techniques that are effective below, e.g. $f_T/10$, may fail for wideband feedback amplifiers.

An additional approach
Simulators should inform the designer whether it is *principally* impossible to compensate for instabilities or not. If not, it is of great value to inform the designer what transfer function is required for additional compensation. It might help to find an additional compensation network that causes the required additional transfer.
In particular, an abstraction level is required that facilitates the separation of physical from non-physical solutions. It must be isolated from the implementation. Practical implementations of compensating networks are usually intertwined with the circuitry. Without loss of generality, we will act as if these networks are fully separable from the forward amplifier and the feedback factor. This may result in an unorthodox description of compensation, however it is wider applicable.

Section 5.2 begins with the de-embedding of the compensation networks from the forward amplifier and from the feedback factor. It summarizes basic properties that result from the restriction that the compensation be passive.
Particular attention is paid to the transfer properties, with no more than a weak link to practical circuits. The major part of the implementation problem is left to the reader and to the referred publications.

5.2.1. Profiled compensation techniques in the forward amplifier

The application of compensation to forward amplifiers (A) and feedback factors (β), is far more effective than modification of the input and output conversion factors (ε and ν). The two forms are identified[2] as *profiled* and *phantom* compensation respectively (see figure 5.1 and the associated subsection).
The name profiled refers to the property that all profiled compensation zeros come through in the effective-aperture at the same location. This property does not hold for the phantom compensation zeros.

The transfer function of a physical network has at least as many poles as zeros, however in feedback loops the number of poles is always higher. It means that the loop gain goes to zero for infinity frequencies, since all physical devices (media) will eventually block all signal flow when frequency increases, if necessary up to lightwave frequencies.
The phase shift, associated with these poles, causes the loop to resonate at distinct frequencies or to oscillate. This is particularly true for the frequency band in which the loop gain approaches the value one. The more poles in the passband the more trouble they cause.

The phase shift, associated with a single pole, is insufficient to cause the loop to resonate. Compensation is therefore required for higher order loops. The same applies for lowband pole-zero *pairs*, because they add no phase-shift to the gain at highband frequencies. The location of midband and highband poles and zeros is critical.
Profiled compensation is basically the addition of zeros in the loop to cancel out the annoying poles in the passband. Because the addition of compensating zeros comes in pairs with new poles, the uncompensated pole is replaced in practice by a new pole positioned at a more favorable location. This is only effective when the new pole position is associated with an optimal trade-off between minimal phase shift and optimal preservation of loop gain. As a result, the essence of profiled compensation is to cover (midband) poles with compensation zeros, and then to add (highband) poles that are precisely positioned at a more favorable location.
The exact requirements for the location of these poles and zeros are discussed in section 5.3; this subsection restricts itself to the overall compensation principle.

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[2] The word *profiled* has been introduced in this text. The word *phantom* is commonly used.

Generic magnitude response

Based on the assumption that in the bandwidth analysis all amplification effort is put in the design of the forward amplifier, the profiled compensation will most likely be passive. An optimal passive profiled compensator $H_{cs}(j\omega)$ must conform to the following requirements:

- It preserves all gain at the high-end of the passband to preserve the available bandwidth. This means that $H_{cs}(j\omega) \rightarrow 1$ in the high frequency band.
- It preserves all gain at the low-end of the passband because modification of a frequency band with high gain has nothing to add to the overall stability. This means that $H_{cs}(0) \rightarrow 1$ in the low frequency band

As a result, an optimal profiled compensator is an attenuator of mid-band frequencies only. A near-optimal profiled compensator causes additional attenuation at low-band frequencies. This is illustrated in figure 5.4.

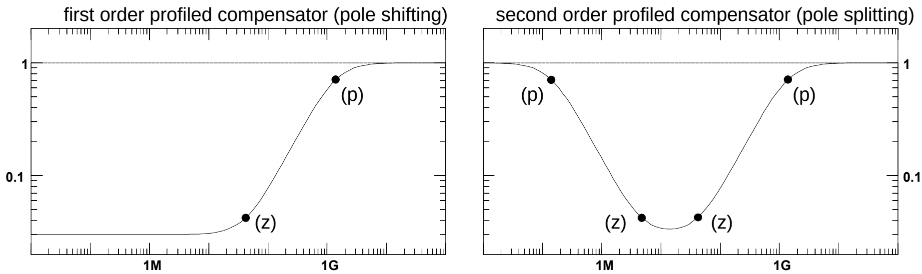

Fig 5.4 *Typical example of the magnitude response of a passive profiled compensator that is cascaded with the forward amplifier. Midband frequencies are attenuated to realize the compensating zeros. All high-band frequencies are passed to preserve the available bandwidth. Attenuation of low-band frequencies deteriorates the feedback accuracy, moderately affecting the passband of the effective-aperture.*

First order profiled compensators, although providing effective compensation, are unable to preserve the gain for low-band frequencies. At minimum, a second order transfer function, or higher, is required to preserve the gain at both ends of the passband. The higher the transfer order, the better the compensation may be.

Another aspect illustrated in figure 5.4 is that it is a bad policy to position the poles, associated with the compensation, at a corner frequency that is *significantly* higher than the bandwidth limit. This causes a reduction of loop gain at high-band frequencies and with that a reduction of available bandwidth.

Generic pole-zero pattern

The magnitude response in figure 5.4 of a second order profiled compensator is typical for optimal higher order profiled compensators: all-pass functions with a dip at mid-band frequencies. This implies that the number of compensating zeros is the same as the number of poles belonging to them.

Further, the gain is approximately one, for frequencies exceeding the bandwidth limit. This implies that the product of all poles equals the product of all zeros.

The position of the poles and zeros is critical. It happens that the zeros coincide with the poles in the special case that *all* singularities in the loop are compensated. This means that the transfer order of the compensating network equals the transfer order of the uncompensated loop. As a result, the overall system order will not increase when employing full compensation. In the more general case where the compensation is restricted to the *dominant* singularities, the zeros will not necessarily coincide with the poles.

5.2.2. Profiled compensation techniques with intertwined solutions

From a mathematical point of view, the profiled compensation is virtually concentrated in a single equalizing network, as though it is cascaded with the forward amplifier. In a practical implementation, the profiled compensation is distributed and embedded in the amplifier stages. The transfer of an uncompensated amplifier stage and its compensation are then intertwined by proper modification of the original transfer function.

There are no fundamental reasons to avoid equalizers as profiled compensators, however from a practical point of view intertwined solutions are more attractive due to their simplicity. Figure 5.5 and 5.6 show examples of intertwined solutions, that are commonly used.

This study has led to increased insight into the principal restriction of intertwined solutions. They modify the uncompensated transfer function, which implies that the uncompensated poles are moved from their original position. This is equivalent to a profiled compensator that matches some compensating zeros with uncompensated poles. As a result, intertwined solutions have fewer degrees of freedom than distinct solutions. An intertwined solution may be sub-optimal, when the presence of parasitic singularities restricts the compensation to a dominant form.

Intertwined compensation by pole shifting

Figure 5.5 shows an example of an intertwined first order profiled compensator. An additional resistor competes with the, mainly capacitive, input impedance of a transistor and absorbs the current gain at low-band frequencies. The asymptotic Bode plots illustrate that this intervention replaces the uncompensated transfer with a mid-band pole by a compensated transfer with a pole at a higher corner frequency. This is called *pole shifting*.

The compensation transfer function is a virtual transfer function that originates from the ratio between compensated and uncompensated current gain. Note that the pole-zero pair of compensation zero and uncompensated pole is available in this virtual transfer function but is absent in the actual current gain.

Practical implementations will always suffer from some degradation of high frequency gain. In this example the addition of a well-adjusted inductor, in series with the absorbing resistor, will relax its absorption at high-band frequencies. Such an inductor will improve the loop gain at higher frequencies.

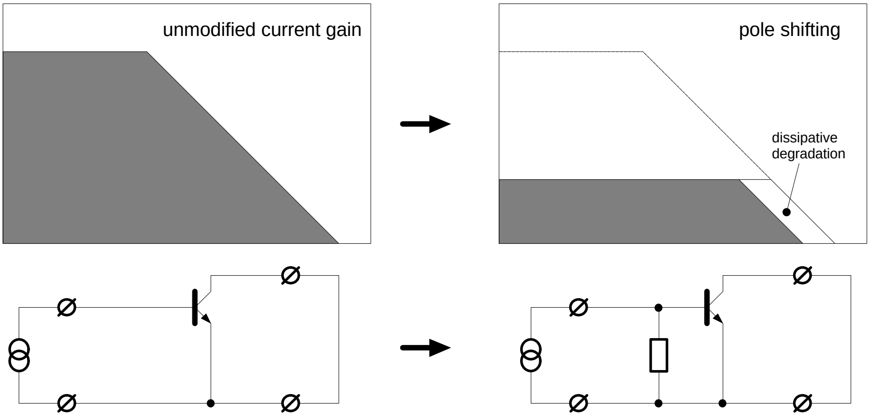

Fig 5.5 *Example of an amplifier stage with intertwined first order profiled compensation. The two asymptotic Bode plots of the current gain illustrate symbolically that the compensated transfer originates from a virtual cascade of the uncompensated transfer and the desired compensation function.*

Intertwined compensation by pole splitting

Figure 5.6 shows an example of an intertwined second order profiled compensator. The additional resistor of figure 5.5 is replaced by a series RC-network for maintaining the current gain at low frequencies. The asymptotic Bode plots illustrate that this intervention replaces the uncompensated transfer with a mid-band pole by a compensated transfer with two poles and a zero. The poles are positioned at either sides of the pass-band and the zero somewhere between these two. This is called *pole splitting*.

The preservation of low frequency gain is one of the benefits of pole splitting, compared to pole shifting. Another benefit is that the position of the generated zero is a free choice. This reduces the number of compensation zeros that is deemed to take the place of a pole, which is a typical restriction of intertwined solutions. The location of no more than one compensation zero is committed to the position of an uncompensated pole.

The second example in figure 5.6 performs an equivalent transfer function with local feedback. The benefit of this approach is that the preservation of the high frequency current gain may be achieved without inductor. A small capacitor, shunted with resistor R_2, relaxes the local feedback for high frequencies, and in this way relaxes the reduction of high frequency gain. The use of a parasitic transistor capacitance, and a properly matched local feedback network may fulfill all requirements.

The launch of similar local feedback loops with both inductors and capacitors facilitates the construction of higher order compensators with complex poles and zeros.

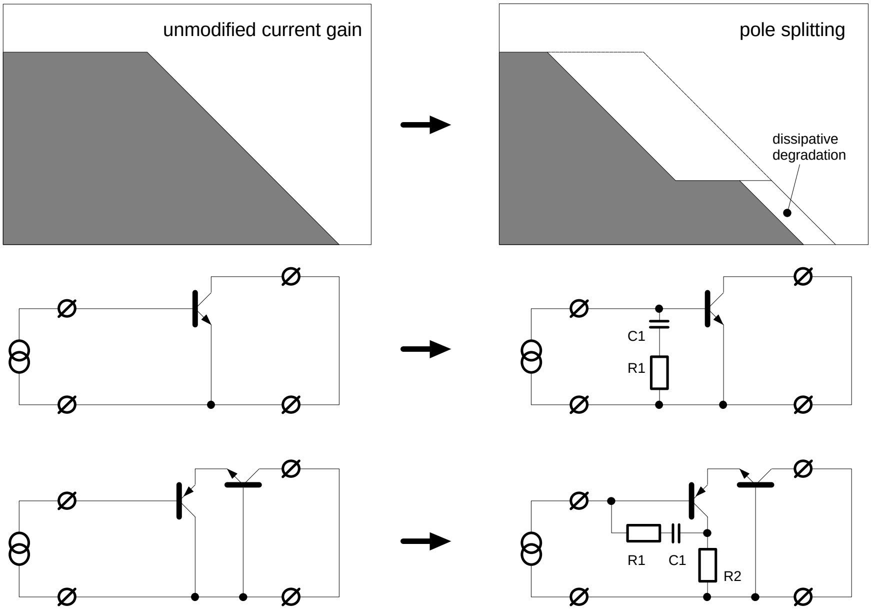

Fig 5.6 *Example of amplifier stages with intertwined second order profiled compensation. The two asymptotic Bode plots of the current gain illustrate symbolically that the compensated transfer originates from a virtual cascade of the uncompensated transfer and the desired compensation function.*

5.2.3. Phantom compensation techniques in the feedback network

The feedback factor is another parameter in the superposition model that is accessible for effective compensation. This type of compensation is identified as *phantom* compensation.

The name *phantom* refers to the property that the insertion of new phantom zeros do not affect the position of the zeros in the effective-aperture and do not expand the number of zeros. This remarkable feature facilitates a way to manipulate effective-aperture with the addition of phantom zeros to the loop but without the addition of new singularities to the effective-aperture.

To make this statement stick, the effective-aperture is written out in polynomial form:

$$\mathring{H}(s) = H_o(s)\cdot H_{cs}(s)\cdot H_{cp}(s) = \frac{T_0(s)}{N_0(s)}\cdot\frac{T_{cs}(s)}{N_{cs}(s)}\cdot\frac{T_{cp}(s)}{N_{cp}(s)} = \textit{loop gain}$$

$$H_{sa}(s) = \frac{1}{H_{cp}}\cdot\left(\frac{\mathring{H}}{\mathring{H}-1}\right) = \frac{T_0\cdot T_{cs}\cdot N_{cp}}{T_0\cdot T_{cs}\cdot T_{cp} - N_0\cdot N_{cs}\cdot N_{cp}} = \textit{effective-aperture}$$

Phantom zeros, which are the roots of the polynomial $T_{cp}(s)$, contribute to the denominator of the effective-aperture however are conspicuous by absence in the numerator. Note that this does not hold for the poles of a phantom compensator.

Phantom compensation is basically the addition of zeros in the loop to affect the undesired poles in the passband. The exact location of these zeros is discussed in section 5.3, however in general they are located at the high end of the passband. This subsection restricts itself to the overall principle.
Unlike profiled zeros, the addition of (passive) phantom zeros is not necessarily accompanied with attenuation of the loop gain. This is because the feedback network is commonly an attenuator itself, and the addition of zeros is nothing more than relaxing this attenuation.
Figure 5.7 shows the impact of this statement. The first example shows a feedback factor of β=1/25, cascaded with a first order (passive) phantom compensator. The second example shows an equal feedback factor, but cascaded with two (complex) phantom zeros. The phantom zeros causes the magnitude response to increase with frequency until all feedback is maximized. As a result, the phantom zeros come in pairs with parasitic poles, and these poles do not improve the overall transfer. The more phantom zeros are inserted, the sooner this limit is reached and the lower the corner frequencies will be of the parasitic poles.
The most conspicuous aspects of phantom compensation are the lack of any loop gain reduction and the ability to keep all parasitic poles out of the passband. Because of these features, phantom compensation is preferred to profiled compensation.

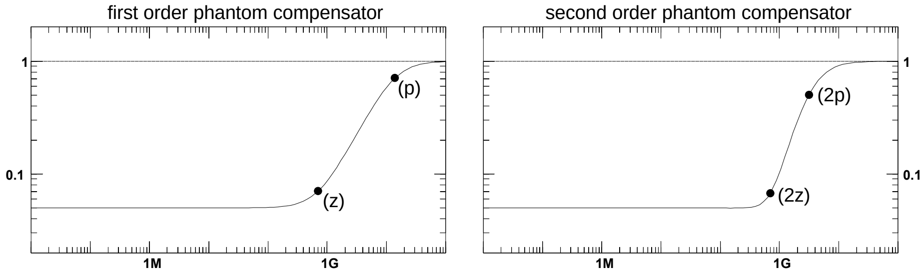

Fig 5.7 *Typical example of the magnitude response of a feedback network with (phantom) compensation. Lowband and midband frequencies are not affected, preserving the intended feedback factor. Since the feedback network is basically an attenuator, the compensator reduces the attenuation for highband frequencies to create the compensation zeros. Note the parasitic pole(s) that occur when all attenuation is neutralized.*

Figure 5.8 shows various examples with phantom compensation in optical receivers. They are all terminated with a capacitive load, because a first order approximation of a practical load is more often capacitive than inductive.
The shunt capacitor, added to the transimpedance receiver in figure 5.8a, increases the RF-signal flow from output to input. This increases the feedback factor and yields a phantom zero.
The series resistor, added to example 5.8b, inhibits the RF-signal flow from the output of the amplifier to the load impedance, which is another way to increase the feedback. The combination of both the serial and the shunt resistor generates two phantom zeros.

In very wideband receivers, the parasitic shunt capacitor across the transimpedance feedback resistor causes a feedback factor with an annoying corner frequency inside the passband. The feedback network in figure 5.8c equalizes the feedback factor by a counter-productive RC-network. The serial resistor, that is added to this example, obstructs this equalization, and facilitates a phantom zero.
The last example in figure 5.8d is an optical receiver with current-current feedback, in which the current flow through the two floating output terminals is fed to the input by a capacitive current divider. This current division is frequency independent. The serial resistor in the grounded portion of this divider causes an unbalanced current division for high-band frequencies and yields a phantom zero. An additional inductor causes two complex phantom zeros.

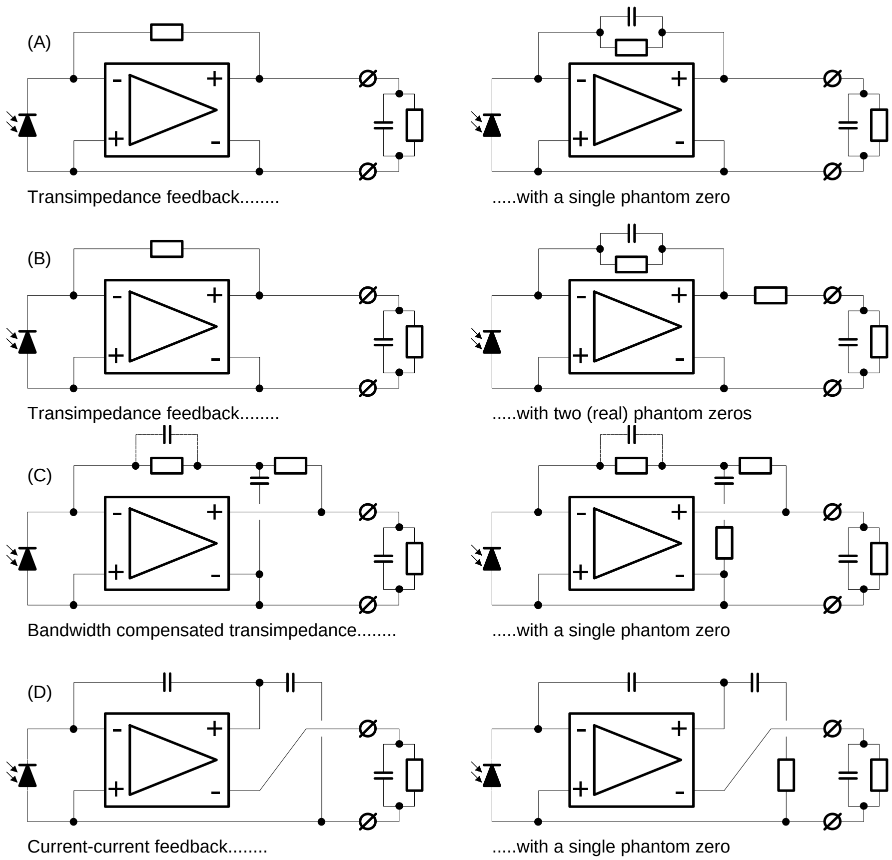

Fig 5.8 *Examples of phantom compensation in various types of optical receivers. All uncompensated examples are drawn on the left side and the associated compensated examples on the right.*

All examples of figure 5.8 suffer from parasitic poles in the transfer of the phantom compensation. The capacitive load and the parasitic feedback capacitors prevent the phantom compensator from being optimal, and introduce an additional penalty. This penalty is configuration dependent and limits the launch of phantom compensation in a particular situation.

5.2.4. Conclusions

In conclusion, some restrictions of optimizing circuit simulators are discussed, when used for compensating feedback amplifiers. A formal compensation synthesis is proposed, that is isolated from how the compensation should be implemented. It enables future circuit simulators to inform designers what compensation transfer is required for proper compensation.

The properties of passive compensation networks in the forward amplifier (profiled compensators) are analyzed. The transfer of optimal (passive) profiled compensators is characterized by a 'dip' at midband frequencies and unity gain at low and high frequencies. The exact transfer is discussed in section 5.3.

Profiled compensators are usually intertwined with the remaining circuitry. These solutions, however, have fewer degrees of freedom than distinct solutions. Some intertwined solutions are summarized using well-known pole-shifting and pole-splitting techniques.

The properties of passive compensation networks in the feedback network (phantom compensators) are analyzed. The transfer of an optimal (passive) phantom compensator is characterized by unity gain at low and midband frequencies, and high gain at high frequencies. As a result, adequate passive solutions are always intertwined with the feedback circuitry.

5.3. Compensation synthesis for feedback amplifiers

The previous section analyzed the transfer of optimal profiled and phantom compensators. This section 5.3 reverses this process, and synthesizes the compensation transfer required to achieve a effective-aperture with an optimal transfer.
The synthesis of the compensation, starting from a specified effective-aperture and available loop gain, is a complicated process. Innumerable solutions do exist but they are very difficult to find on a trial and error base.
Nordholt [406] proposed a structured method to design the compensation for the special case that the loop gain is an all-pole transfer function with two or three poles. Stoffels [510] automated this design method, to generate a circuit diagram and its compensation, guided by user specifications. Essential for the above methods is that the annoying effects of parasitic poles and zeros is adequately reduced by additional sub-circuits (buffering, cascoding).
At low frequencies, e.g. below $f_T/10$ for second order loops, this approach is successful. A fully automated synthesis of amplifiers has been demonstrated [510] for frequencies up to 1 MHz (second order loop). However, these methods may fail for increasing bandwidth demands, e.g. above 200 MHz.

Section 5.2 discussed the use of circuit optimizers, to extend the applicability of these structured methods. However, a serious problem arises when optimizers are as well ineffective in achieving the desired frequency response.
These wideband feedback amplifiers have in common that they suffer from loops with dominant zeros, from RHP-zeros and from many parasitic poles and zeros that cannot be ignored. The order of the loop transfer function is then too high for compensation of *all* relevant singularities.
No solutions were found in the literature on controlling these loops.

This section 5.3 introduces a powerful algorithm for compensation synthesis that has been developed during this study. It provides the transfer function that is required for additional compensation.
The proposed method can handle loops with dominant RHP-zeros and loops that suffer from parasitic singularities. This algorithm predicts the required compensation, in terms of poles and zeros, where conventional (manual) methods fail.
Finally, examples are presented in this section that illustrate the power of the synthesis algorithm, and may serve as a template for manual synthesis.

5.3.1. Bandpass synthesis

Consider a feedback amplifier topology, without stability compensation, for instance designed using Nordholt's method [406]. A first requisite for automated compensation synthesis is a mathematical description of the desired effective-aperture passband. This mathematical description requires the specification of the *generic passband* and the evaluation of the *specific passband.* Note that an unambiguous passband specification is not equivalent to a unique compensation solution.

generic passband specification

Compensation synthesis starts from the specification of a generic passband. The generic passband, is a normalized transfer function that serves as a scale model for the effective-aperture. Its dc-gain and its asymptotic corner frequency are one by definition. Further, the generic passband is defined to be a minimum-phase transfer function, which means that all singularities are positioned in the left complex half-plane.

A practical effective-aperture is a low-pass transfer function that must be smoothed and optimally flattened. Non-coincident pole-zero pairs are undesirable in the generic passband because they introduce additional ripple. The generic passband is therefore primarily an all-pole transfer function, whose order is chosen as low as possible.

Select a suitable all-pole transfer function, for instance a Bessel, a Butterworth or a Chebyshev transfer function, based on the bandpass analysis presented in section 5.1.3. The pole-zero patterns in figures 5.9a and 5.3 are typical for these transfer functions.

specific passband evaluation

The specific passband serves as a properly dimensioned model for the effective-aperture. The specific passband is evolved from the generic passband through appropriate magnitude, bandwidth and delay. The scaling operations facilitate matching the effective-aperture to the design goal, with respect to the compensation limits of the loop.

The specification must take account of the limitations of passive compensation, so that the specified bandwidth should match the available loop bandwidth. We identify this as magnitude and bandwidth scaling. One way to modify the feedback loop to match this passband specification is removing the undesirable poles by introducing coincident compensation zeros.

Although the opposite applies for LHP-zeros, RHP-zeros cannot be compensated using RHP-poles due to stability considerations. This study has resulted in novel techniques on handling RHP-zeros by treating them as *pseudo delay*. Building on this concept, a method has been developed in this study that adds an appropriate delay to the passband specification to satisfy the compensation limitations of the loop. We identify this as delay scaling.

Let us recall what modifications to the specific passband are possible that preserve the magnitude and bandwidth scaling. A typical passband specification is illustrated in figure 5.9.

- Figure 5.9a is indicative for the initial pole-zero pattern of the generic passband, resulting from appropriate scaling in magnitude and bandwidth.
- The addition of coincident pole-zero pairs, as in figure 5.9b, will not affect the transfer properties. Therefore, they are allowed in a passband specification when required.
- Addition of mirrored pole-zero pairs, such as in figure 5.9c, contributes a frequency-dependent phase shift. This is equivalent to pseudo delay within a limited frequency band. These pole-zero pairs are allowed in a passband specification when unavoidable and when their magnitude is equal or higher then the width of the passband.

All other pole-zero patterns are in principle undesirable.

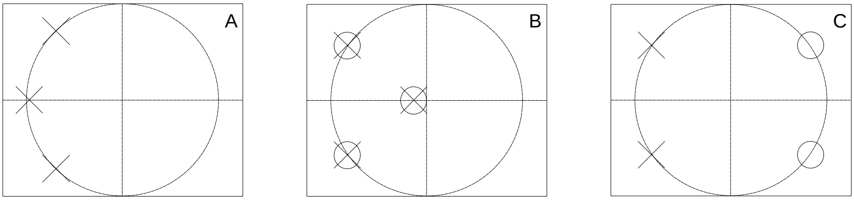

Fig 5.9 *Example of the basic pole-zero patterns that composes the specific passband of the effective-aperture. It comprises of: (a) a low-pass all-pole transfer function, (b) an all-pass transfer function with coincident pole-zero pairs and (c) an all-pass phase-shifting transfer function with mirrored pole-zero pairs.*

What pole-zero pattern is most appropriate for meeting the passband specification? From the definition of the effective-aperture it is observed that the zeros in the profiled loop gain show up at the same location as in the effective-aperture. Since the specific passband is preferably an all-pole transfer function, these zeros call for cancellation by coincident poles. They should not pop up in the passband specification, or at most in the shape of a coincident pole-zero pair, as drawn in figure 5.9b.
Unfortunately, this is not feasible for RHP-zeros. We assume that the best approach for dealing with RHP-zeros is to let them function as pseudo time delay. They should therefore appear in the passband specification in combination with poles that are mirrored with respect to the imaginary axis, as illustrated in figure 5.9c.
In short, all zeros in a compensated effective-aperture must appear in the passband specification, in combination with poles at their *equivalent minimum phase* position. These poles are defined as poles that create coinciding pairs with LHP-zeros and create mirrored pairs with RHP-zeros. The mirrored pole-zero pairs contribute a constant phase delay to the effective-aperture over a wide frequency interval. The coinciding poles and zeros neutralize each other.
From the definition of the effective-aperture it is recognized that the poles in the loop gain show up at a different location as in the effective-aperture. They represent the remaining restrictions on the feedback loop, and there is nothing left than to accept these poles in the specification. The best that can be achieved is to position them as in the generic passband, properly scaled in bandwidth.

All scaling steps together result in the following transformation of generic passband into specific passband:

- *Magnitude scaling* contributes an exact match with the effective-aperture at zero frequency. This scaling factor can be evaluated simply. It equals (g)/(g-1), in which g is the dc-gain of the loop.
- *Bandwidth scaling* contributes a close magnitude match to the effective-aperture for all frequencies within the passband. It requires the calculation of the available bandwidth (ω_0), starting from a dominant magnitude match for infinite frequencies. This calculation has been described in section 5.1.2.
- *Delay scaling* is the finishing touch and contributes a full match of both magnitude and phase. It fits the minimum-phase transfer of the generic passband, and the non-minimum-phase transfer of a effective-aperture with RHP-zeros. When $T_0(s)$

represents the polynomial with all profiled zeros of the loop gain and $\mathcal{T}_0(s)$ represents the associated polynomial with all roots at their equivalent minimum phase position, than the transfer function $T_0(s)/\mathcal{T}_0(s)$ provides the (pseudo) delay requested.

The final specification of the specific passband A(s) results in the following mathematical form:

$$A(s) = \frac{g}{g-1} \cdot \frac{T_0(s)}{\mathcal{T}_0(s)} \cdot \frac{1}{B_0(s/\omega_0)} \qquad = \textit{specific passband}$$

In this expression, the polynomial $B_0(s)$ refers to the numerator of the chosen generic passband. The quantity g refers to the dc-loop-gain and performs the magnitude scaling. The quantity ω_0 refers to the available bandwidth and performs the bandwidth scaling. The polynomial $T_0(s)$ refers to the denominator of the profiled loop gain, and its roots produce the zeros of the effective-aperture. The polynomial $\mathcal{T}_0(s)$ refers to the minimum phase equivalent of $T_0(s)$, and its roots produce the poles in the effective-aperture that accompanies the zeros. The rational function $T_0(s)/\mathcal{T}_0(s)$ forms an all-pass transfer function with a pseudo delay component.

5.3.2. Compensation algorithm

A unique solution for the compensation of a feedback loop does not exist. Nevertheless, an algorithm should lead to a unique result. In this section, the prior conditions are established that will yield the desired unique solution, which may be linked to practical compensation considerations.

Next, a powerful compensation algorithm is introduced, in two steps. The *full* compensation algorithm holds for the special case that all poles and zeros in the loop are compensatable. The *dominant* compensation algorithm generalizes the synthesis for loops with parasitic poles and zeros.

prior conditions for a unique compensation solution

Consider the synthesis of a particular loop gain requirement, starting with an n^{th}-order specific passband transfer function. The synthesis procedure should find an expression for the loop gain matching with (n+1) constants, for instance organized as the poles and the dc-gain of the predefined specific passband.

Reliable loop gain synthesis starts from the assumption that the desired specific passband is compatible with the dc-gain and the available bandwidth of the loop. The reduced synthesis should match with the (n–1) remaining constants, for instance organized as the parameters of the generic passband transfer function. In practical terms: the compensation of an n^{th}-order loop requires at least (n–1) compensation interventions. The loop gain has at least n poles in the loop, and that is one more than is required for a unique solution. The degrees of freedom increase in number when (phantom) zeros are permitted in the solution for a compensated loop gain. On one hand, it is not difficult to find just some solution, however a practical implementation of this solution will not fulfill the requirements of practical feedback loops. On the other hand, it is very difficult to find the remaining unique solution using manual synthesis, imposed by prior conditions.

The synthesis algorithm proposed here exactly solves (n–1) loop parameters in the case that all phantom zeros (m) in the feedback network and some poles (n–m–1) in the profiled loop gain are variable. This number is the exact number that is required to yield a unique compensation solution.
All remaining loop parameters are arbitrarily predefined, which is equivalent to fixing all zeros and all remaining poles in the uncompensated loop gain. These prior conditions link up nicely with practical compensation problems because most singularities in the loop maintain their position after a successful sequence of compensation interventions.

full compensation
Initially, the question which loop gain $\mathring{H}(s)$ yields a effective-aperture with predefined generic passband is considered in the case that dc-gain of the loop and some poles and zeros are predefined? To find this solution, the compensated and uncompensated loop gain are described in the following rational form:

$$H_0(s) = g \cdot \frac{T_0(s)}{N_{00}(s) \cdot N_{0x}(s)} = g \cdot \frac{T_0(s)}{N_0(s)} \qquad = \textit{uncompensated loop gain}$$

$$\mathring{H}(s) = g \cdot \frac{T_0(s) \cdot T_x(s)}{N_{00}(s) \cdot N_x(s)} = g \cdot \frac{\mathring{T}(s)}{\mathring{N}(s)} \qquad = \textit{compensated loop gain}$$

The polynomials $N_0(s)$ and $T_0(s)$ represent the uncompensated poles and zeros and the constant g represents the dc-gain of the loop. All zeros (T_0) and some poles (N_{00}) are predefined, which means that the compensation may not modify the position of these singularities. The remaining poles (N_{0x}) are selected for modification by the compensation procedure.
The polynomials $\mathring{N}(s)$ and $\mathring{T}(s)$ represent the compensated poles and zeros, of which $N_{00}(s)$ and $T_0(s)$ are the predefined singularities, and $N_x(s)$ and $T_x(s)$ are the unknown singularities that will be extracted by the synthesis process. *All* roots of $N_x(s)$ are originated in the forward amplifier (profiled poles) and *all* roots of $T_x(s)$ are compensation zeros in the feedback network (phantom zeros). All polynomials are normalized in magnitude to ensure that $N_{00}(0)=T_0(0)=1$ and $N_x(0)=T_x(0)=1$.
Appendix *J* describes the mathematics of the algorithm proposed for the extraction of the polynomials $N_x(s)$ and $T_x(s)$. It is based on the weighted polynomial division of appendix *G*, that is an integral part of this study. A symbolic representation for this process, annotated as a function of four input variables and two output variables, yields:

$$[N_x(s),\ T_x(s)] = \textit{CompSynthesis}\,(g,\ N_{00}(s),\ T_0(s),\ B_0(s/\omega_0))$$

It requires the generic passband polynomial $B_0(s)$, scaled in bandwidth by the available bandwidth ω_0 of the uncompensated loop gain. The order (m) of $N_{00}(s)$ is the number of profiled poles (m) that must maintain their positions, and this presets the required number of phantom zeros (m) that will be produced by the output polynomial $T_x(s)$. The order (n) of $B_0(s)$ fixes the number of profiled poles (n–1–m) that must be modified and that will be produced by the output polynomial $N_x(s)$. The other two input parameters specify the predefined poles and zeros and the loop gain.

The two extracted polynomials, $N_x(s)$ and $T_x(s)$, produce the transfer functions of the compensating networks. From the definition of compensated and uncompensated loop gain, we obtain:

$$H_{cp}(s) = \frac{T_x(s)}{1} = \textit{phantom compensator, with order (m)}$$

$$H_{cs}(s) = \frac{N_{0x}(s)}{N_x(s)} = \textit{profiled compensator, with order (n-m-1)}$$

The zeros in the profiled compensator cancels all the selected poles (N_{0x}) of the loop. This is because $N_{0x}(s)$ originates from the uncompensated loop. In practice, this is equivalent to the replacement of some poles in the loop, by some new, precisely positioned, compensation poles in $N_x(s)$.
Furthermore, the algorithm ensures that some of the remaining poles in the profiled compensator are used to cover all LHP-zeros of the uncompensated loop. This minimizes the effective order of the compensated loop gain.
An inaccurate evaluation of the available bandwidth causes a difference between the highest coefficients of the polynomials $N_{0x}(s)$ and $N_x(s)$. This is because $N_{0x}(s)$ is not an input parameter for the algorithm. As a result, the better that ω_0 is estimated, the better that $H_{cs}(s)$ approximates one for infinite frequencies.

dominant compensation

Up to now, it has been understood that all the loop singularities are compensatable. This implies that all singularities are within or close to the bandwidth circle, and that the compensation solution will be exact. Restrictions to the zeros of the intertwined compensation solutions are no longer of further concern with full compensation, since all profiled zeros coincide with uncompensated poles in the loop.
Full compensation is feasible when parasitic singularities are irrelevant, which is typical of low frequency design. In the case of significant parasitic singularities, a dominant approximation of the loop is required to keep control over the transfer order of the compensation solution. This requires the replacement of all singularities by a smaller set with (other) singularities that approximate the loop within a predefined frequency band. Thereafter, apply the overall compensation synthesis to the dominant singularities and proceed as described above.
The necessity to correct the phase of the loop gain is concentrated near the maximum available bandwidth ω_0. This implies that the accuracy of the approximation must be optimized near ω_0. Start therefore from the estimation of the available bandwidth ω_0 and perform the dominant approximation from dc to ω_0. Especially the refinement step of the accuracy near ω_0, as described in section 4.5.3, is important because accuracy failures will significantly deteriorate the overall synthesis result.
The introduction of phantom zeros by the synthesis algorithm results in additional parasitic poles in the loop. Their location is application dependent and is discussed in section 5.2.3. Since they were ignored in the synthesis, an iterative approach based on our synthesis procedure is required:

- Initialize the compensation synthesis based on an uncompensated loop.
- Estimate the additional parasitic poles associated with phantom compensation.
- Add these poles to the uncompensated loop and recalculate the compensation.

Repeat these iterative steps as often as is required to stabilize the predicted results.

A practical implementation of the compensation prefers the use of intertwined compensation networks. Unfortunately, the compensation zeros required do not coincide with the (primary) poles in the loop, when applying dominant compensation. Instead, they coincide with those dominant poles that differ in position from the primary poles. As a result, the use of intertwined compensation schemes may restrict or in fact undo the compensation performance. This is because the intertwined compensation solutions are directly associated with compensation zeros coinciding with primary poles.
A practical solution to this problem is a complementary mathematical adjustment of the predicted compensation transfer function. This adjustment step must modify all compensation singularities to force one or more profiled compensation zeros to a predefined position and to conserve a fair approximation of the required transfer function.
This is illustrated in the following example of a third order profiled compensator. The right side of the equation is the original result that is predicted by a dominant compensation synthesis and the left side is the modified result

$$\frac{(1-s/z_{10})\cdot(1-s/z'_2)\cdot(1-s/z'_3)}{(1-s/p'_1)\cdot(1-s/p'_2)\cdot(1-s/p'_3)} \approx \frac{(1-s/z_1)\cdot(1-s/z_2)\cdot(1-s/z_3)}{(1-s/p_1)\cdot(1-s/p_2)\cdot(1-s/p_3)}$$

In this example, all singularities are modified to change the zero z_1 into z_{10} and to facilitate a fair agreement between the left hand expression and the right hand expression. This process is equivalent to:

$$\frac{(1-s/z'_2)\cdot(1-s/z'_3)}{(1-s/p'_1)\cdot(1-s/p'_2)\cdot(1-s/p'_3)} \approx \frac{(1-s/z_1)}{(1-s/z_{10})}\cdot\frac{(1-s/z_2)\cdot(1-s/z_3)}{(1-s/p_1)\cdot(1-s/p_2)\cdot(1-s/p_3)}$$

Of course there exist no exact solution and this clarifies the statement that an intertwined compensation may restrict the overall compensation performance. To find a fair approximation, an appropriate deflation technique can be used, such as the pole-zero cancellation algorithm of section 4.5.2. In an emergency, the curve fit algorithm for rational functions can give a solution.
Note that pole splitting techniques are preferred to pole shifting techniques, because they entail more degrees of freedom. Pole splitting techniques make (at least) one additional zero available for compensation, which is not restricted to the location of another (primary) pole in the loop.

5.3.3. Examples of loops with full compensation

Practical feedback design cannot rely on the blind use of automated synthesis tools. Compensation synthesis always requires some manual assistance, and root-locus plots are quite suitable for the assessment of the synthesized result. These plots give information on the loop gain, the effective-aperture, and the manner in which these two are interrelated.

A root-locus plot is a composite description of an infinite number of pole-zero patterns, with a minor difference between them. It is commonly practice to vary the dc-gain parameter from zero to infinity although there is no good criterion for selecting this parameter for the *best* available choice. These plots are useful because they are amenable to *pattern recognition* techniques. The effectiveness of this manual technique is highly dependent on the correlation of interpretative graphic rules for associative root-locus behavior with distinct changes in the loop.
This subsection summarizes various typical examples of root-locus plots to acquaint the reader with a number of commonly known graphic rules based on pattern recognition.

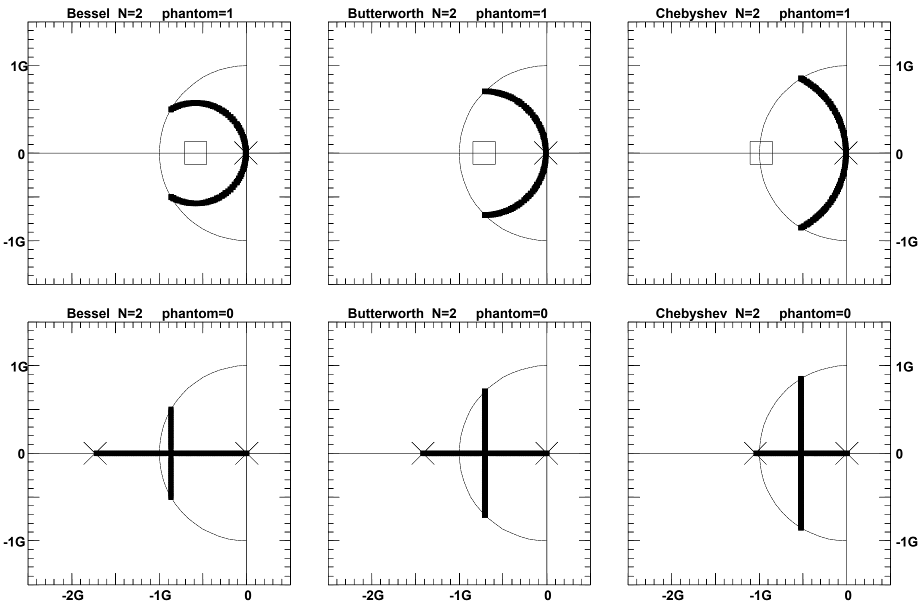

Fig 5.11 *Examples of finite root-locus plots for compensated second order feedback loops with all dominant poles. In these examples, the poles near the origin are at 10 MHz and the feedback bandwidth is 1 GHz.*

Figure 5.10 to 5.12 show various *finite* root-locus plots for a different number of phantom zeros and for various generic passband specifications. Finite root-locus plots are slightly different from the usual infinite root-locus plot. Finite root-locus plots illustrate the positions of all poles and zeros of the virtual-aperture, while the dc-gain is varied continuously from its *current* value down to zero.
The points of the curved lines originate from the poles of the following transfer function, while α varies from zero to one:

$$A'_{la}(s) = \left(\frac{\alpha \cdot \mathring{H}}{\alpha \cdot \mathring{H} - 1} \right) \qquad \textbf{\textit{with}} \qquad \{\alpha \in ® \mid 0 < \alpha \leq 1\}$$

All infinite numbers of poles are marked with dots, which yields the curved lines. The 'beginning' of the lines are marked with cross markers that represent the poles of the loop gain in a usual way. The 'end' of the lines mark the positions of the poles of the virtual-aperture, which is equivalent to the positions of the poles of the effective-aperture. The phantom zeros are represented by square markers, while all remaining zeros (if any) are represented with circular markers.

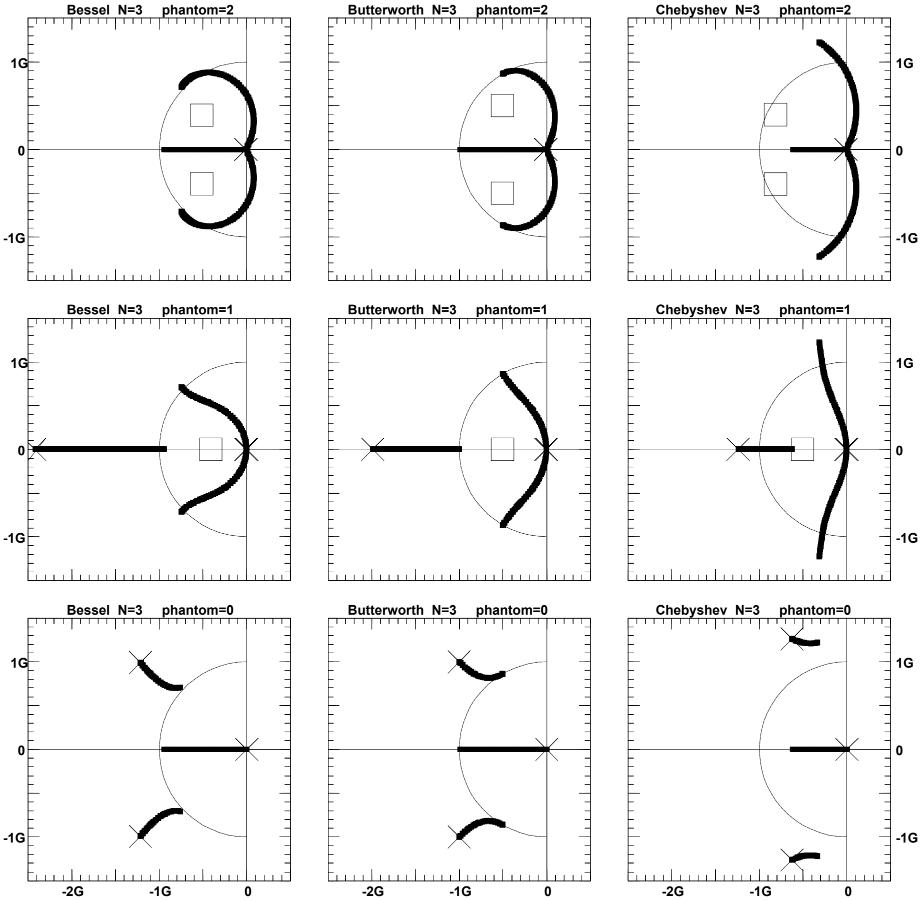

Fig 5.11 *Examples of finite root-locus plots for compensated third order feedback loops with all dominant poles. In these examples, the poles near the origin are at 10 MHz and the feedback bandwidth is 1 GHz.*

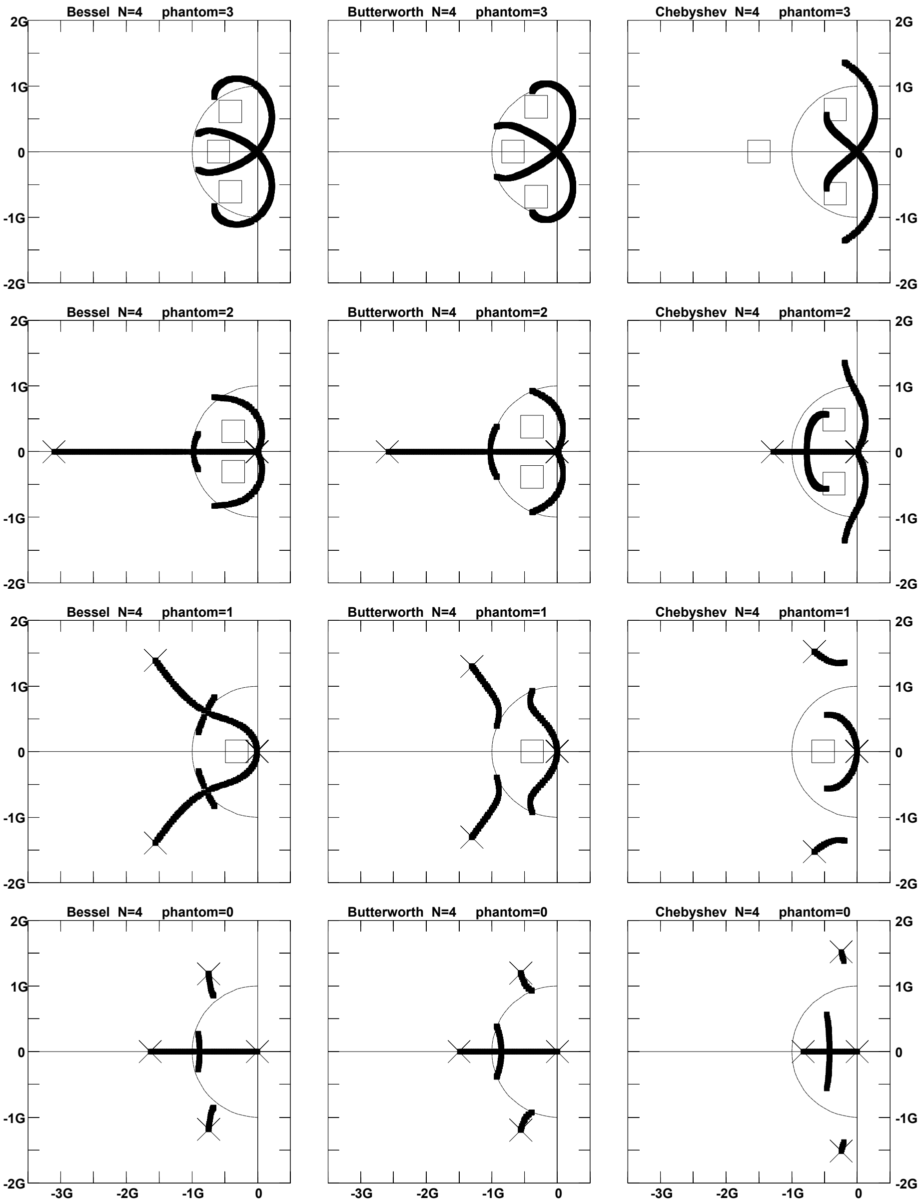

Fig 5.12 *Examples of finite root-locus plots for compensated fourth order feedback loops with all dominant poles. In these examples, the poles near the origin are at 10 MHz and the feedback bandwidth is 1 GHz.*

The pole-zero patterns in figure 5.10 to 5.12 are typical of compensated loops, in which all amplifier stages provide a dominant pole of which the corner frequency is significant smaller than the available bandwidth. These are all idealized situations; an example from practice would never look that perfect.

When plotted linearly, the uncompensated pole positions are close to the origin of the pole-zero pattern, with respect to the radius of the bandwidth circle. In all these examples the corner frequencies of the uncompensated poles are fixed at 1% of the available bandwidth. The exact position is not relevant because poles locate at 10% or 0.1% will result in nearly similar root-loci. The remaining pole positions are adjusted by the compensation, when required.

Various rules of thumb may be extracted from these typical examples. At least the commonly known rules are of value, such as:

- All root loci start from the positions of the poles in the loop.
- All extrapolated root loci end at the positions of the zero in the loop or end in infinity, when the loop gain is expanded to infinity.
- The poles and zeros pass for virtual magnets that 'repel' the root locus lines with poles and 'attract' the root locus lines with zeros.

Further, these plots show that:

- All compensating poles and all phantom zeros are located *near* the bandwidth circle, when a *logarithmic* scale was used.
- Most phantom zeros are located *inside* the bandwidth circle.
- Most compensating poles are located *outside* the bandwidth circle.
- A magnitude response that emphasizes the high end of the passband (Chebyshev low-pass transfer) requires an increase in magnitude for the phantom zeros or a decrease in magnitude for the compensating poles. The converse holds for a passband with a smoothed magnitude response (Bessel).
- Compensation using the insertion of *two* or more phantom zeros requires a complex pair of zeros.
- Compensation by displacement of *two* or more poles requires a complex pair of poles.
- The insertion of *two* or more phantom zeros requires a complex pair of zeros.

These empirical rules are valuable for a first guess on the location where the phantom zeros should be inserted or whereto the poles should be moved.

5.3.4. Examples of loops with dominant compensation

The typical examples in section 5.3.3 were intended to get experience in the interpretation of computer generated root-locus patterns. They are applicable for manual compensation synthesis in a frequency band in which primary poles and dominant poles are almost equal.
Manual synthesis of complex feedback loops, such as the examples in section 4.4.3, is next to impossible, and must rely on automated synthesis. This subsection 5.3.4 demonstrates the compensation of realistic feedback loops.

We start from an arbitrary example of a loop covering three cascoded amplifier stages. In section 4.4.3 three examples of transimpedance feedback amplifiers are discussed to show their associated loop gain as Bode plot and as pole-zero pattern. The loop of example *b* (cascoded amplifier stages) was modeled with six poles and three zeros up to 3 GHz.
At first, the available bandwidth was estimated and rated at 490 MHz. Next, the loop was deflated with a dominant approximation of four dominant poles, and one dominant zero. Because this dominant zero is located in the *right* half plane, further deflation by pole-zero cancellation is out of the question. Finally, the synthesis algorithm is applied to the deflated loop, to produce one of the results of figure 5.13.
Three different compensation solutions were evaluated for a Butterworth passband:

- No phantom zeros and a 4th order profiled compensator
- One phantom zero and a 3rd order profiled compensator
- Two phantom zeros and a 2nd order profiled compensator

The plots in figure 5.13 show that the more phantom zeros are available for use, the less attenuation is required in the profiled compensator, and the lower its transfer order will be. For convenience, the parasitic poles are ignored that are generated in the phantom compensator.

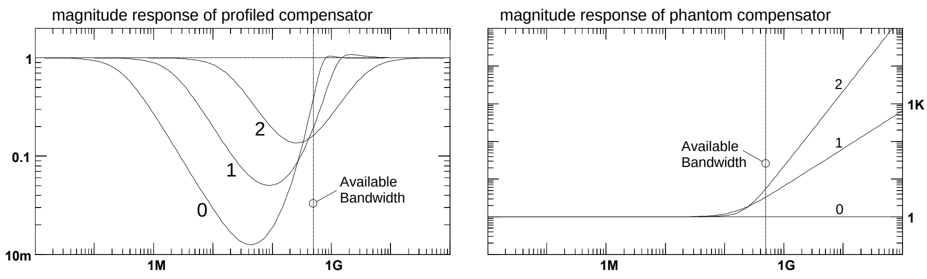

Fig 5.13 *Examples of the magnitude response of the required compensation for a dominant 4th order feedback loop. The numbers 0,1,2 refer to the number of phantom zeros that are inserted in the loop.*

The effective-aperture that arises from the compensated loop is shown in figure 5.14. In this plot, the dominant compensation is combined with all the poles and zeros of the uncompensated loop. As a result, the magnitude responses differ from the specified passband, especially *above* the passband.
Note that the ripple on the deviation from the specified passband increases with the number of phantom zeros. This illustrates that phantom compensation is not *always* preferable to other compensation techniques, as was stated by Nordholt [406:*p175*]. We

emphasize that the efficiency of phantom compensation is deteriorated by its own parasitic poles.

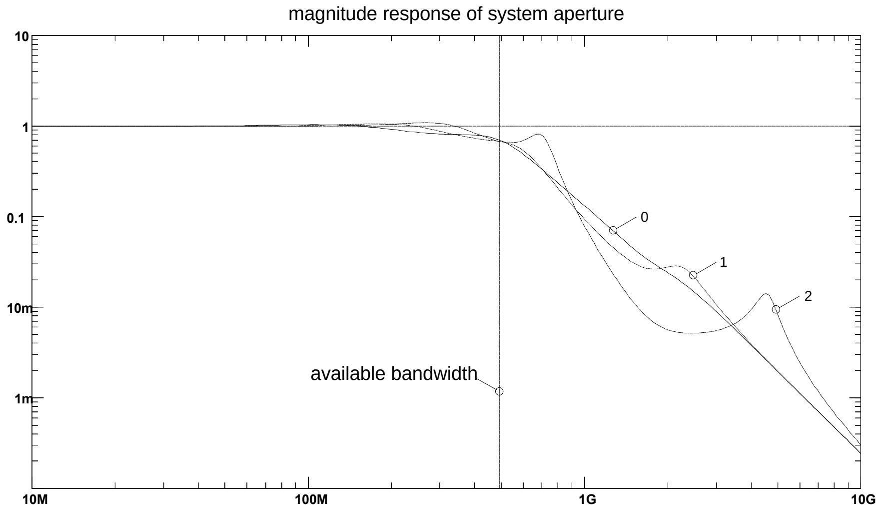

Fig 5.14 *Examples of the effective-aperture that is associated with the compensation of figure 5.13. The numbers 0,1,2 refer to the number of phantom zeros that are inserted in the loop.*

All the feedback loops mentioned here are stable. This is demonstrated in the root-locus plot of figure 5.15. Both uncompensated and compensated loops are plotted, for the situation that one phantom zero is inserted.

The mirrored positions of some poles, with respect to the RHP-zeros, are all out of recognition. This is because those positions would have captured in a modified root-locus plot where the singularities are replaced by their dominant counterparts. The complexity of the current root-locus plot illustrates why manual synthesis will fail for these loops.

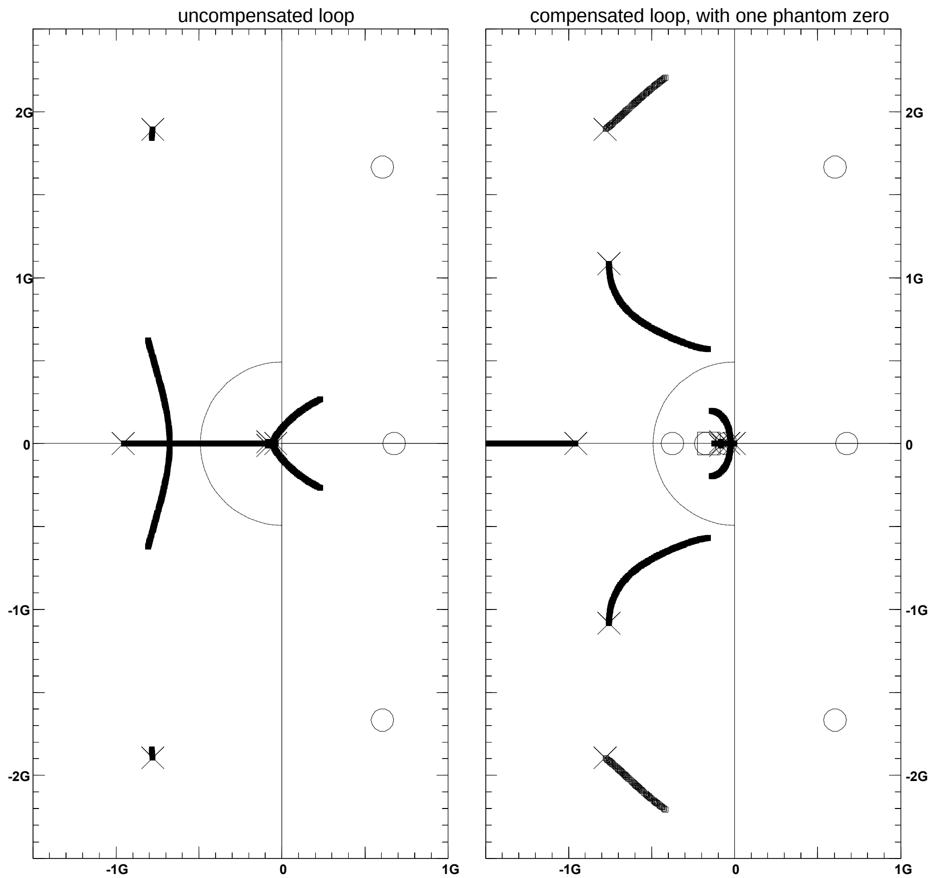

Fig 5.15 *Example of a reliable root-locus plot. The uncompensated loop was modeled with six poles and three zeros. The compensation has added one phantom zero and a third order profiled compensator.*

5.3.5. Conclusions

In conclusion, the difficulties are discussed that arise when the order of the loop transfer function is too high for compensation of *all* relevant singularities. Conventional compensation methods will then be inadequate. The assistance of circuit optimizers may then become crucial, however they require realistic design goals.

A bandpass synthesis algorithm is developed that provides an accurate estimation of the effective-aperture, when optimal passive compensation is applied. This transfer function, the *specific passband*, is extracted from the poles and zeros in the loop using the aperture analyses of section 5.1 and the deflation algorithms of section 4.5.
The bandpass synthesis is robust and is proofed against RHP-zeros in (right half plane). The algorithm let these zeros show up as pseudo delay. This means that the pole-zero pattern of the predicted effective-aperture includes a mirrored pole-zero pair.

The synthesized specific passband is of direct use as input for automated circuit optimization.

A compensation synthesis algorithm has been developed, in terms of poles and zeros. As input, it requires the result of the specific passband calculation and the number of phantom zeros that is required. As output, it provides an accurate estimation of the required transfer functions of the two compensation networks: profiled compensator (in forward amplifier) and phantom compensator (in feedback network).
The designer can use this for assessing the feasibility of the compensation. If it is feasible then it is of direct use in designing the compensation network configuration. This is significant simpler than designing it from scratch.
An optimizer may fine tune the compensating elements for optimal performance.

The performance is demonstrated of the compensation synthesis algorithm, in the exceptional case that *all* poles of the loop are compensated (full compensation). These examples are primarily intended for *pattern recognition* when assessing root-locus patterns.
Furthermore, the performance of the compensation synthesis algorithm is demonstrated in the realistic situation that the loop has many poles and zeros, of which a few of them are dominant (dominant compensation). It demonstrated that the loop of example "b" in section 4.4.3 is compensatable, although higher order compensation networks are required.