Chapter 7

Formal description of noise

The design of high performance wideband feedback amplifiers requires optimization of transfer as well as noise performance. Nordholt [406] developed a design synthesis that handles either design goal. The preceding chapters have dealt with Nordholt's synthesis to feedback amplifier transfer in the case that active decoupling circuitry is ineffective in reducing parasitic effects. The proposed improvements rely extensively on transfer measurements between well-defined reference planes and on the extraction of adequate device models.

State-of-the-art noise measurements are not as accurate and straight forward as state of the art transfer measurements. The application examples in the previous chapter have, therefore, relied on *estimated* transistor noise models. The use of estimated noise models yields in many cases reasonable results. Nevertheless, the design of amplifiers with optimal noise performance requires more accurate models. Accurate noise measurements on lightwave receivers have indicated that simulated and measured noise are poorly correlated. This difference is relevant, for instance, when designing lightwave receivers with *optimal* wideband noise tuning.

Improving the design of low noise amplifiers requires the use of improved noise models extracted from *measured* data. Furthermore, simulation techniques for adequately handling the thermal noise of passive components with parasitic effects, such as lossy inductors in input-tuning networks (see section 6.3.2), are required.

Chapter 7 reviews the mathematical tools for (1) analyzing and measuring noisy circuits and (2) extracting device noise models. Measurement techniques will be discussed in chapter 8 and 9.

State of the art

Many articles have been published on modeling and characterizing semiconductor devices. Microwave applications are often focused on minimizing noise using resonance techniques. The minimization of noise figure in a small frequency band is a prime goal. From this practice, *spot noise parameters* have been evolved, as is described in section 7.3.2. These parameters are dedicated to narrowband noise optimization, however, they are very inconvenient when designing wideband amplifiers. As a result, most publications are focused on spot noise parameters. The same applies to commercially available microwave circuit simulators such as Touchstone©[124], which often interfere with gaining insight into the noise performance of wideband amplifiers.

Measured noise parameters of semiconductor devices are at best sparingly available. An increasing number of manufacturers specify their semiconductor devices in terms of spot noise parameters, however, they are usually restricted to a few frequencies above 1 GHz. These frequency intervals chosen are often too coarse for adequate verification of device noise models.

On the other hand, various models have been published that rely on physical understanding of the devices. Some authors have compared their own models with measured data. These measurements indicate a rough agreement between model and measured noise, and are often restricted to a few frequencies above 1 GHz. When adequate noise measurements at several frequency points below 1 GHz are lacking, there is no way of judging whether parasitic noise effects are of relevance[1] to wideband applications or not.

Highlights

This chapter starts with unambiguous definitions of various signal and noise spectra. Most of the chapter relies on known[2] concepts. The highlights of this chapter, as considered in this study, are:

- Detailed and compact survey of known matrix methods for describing noisy multi-port networks. Voltage-, current- and wave methods are discussed simultaneously, and the wave methods include waves normalized to arbitrary (complex) reference impedances.
- Detailed and compact summary of various two-port noise parameters.
- Generalization of transformation rules between spot noise parameters and the wave correlation matrix. Their validation has been extended to waves normalized to reference impedances with negative real part.
- Introduction of a promising new concept for specifying the noise of semiconductor devices: *autonomous noise parameters*. They may simplify the extraction of transistor noise models from two-port noise measurements.

This chapter forms the theoretical foundation for the noise measurements that are described in chapter 8 and 9.

[1] An example of a parasitic effect of minor importance is induced gate noise in FETs. Nordholt [406] has shown that this effect is of secondary importance in the case of wideband amplifiers. Since adequate measurement data is lacking, it is conceivable that these parasitic effects are small compared to others that have been ignored. This illustrates the importance of using device noise models that are entirely based on measured data.

[2] We emphasize that this chapter reviews in essence known microwave concepts. Nevertheless, any summary perusal of the plethora of microwave texts should be sufficient to illustrate the fact that in general these texts survey an incomplete overview of the subject, with restricted validation of the formulae. Some texts use differing definitions among confusing typographical errors. Many textbooks and articles had to be consulted to complete this compact overview.

7.1. Definitions of signal and noise spectra

Section 7.1 considers unambiguous definitions for various signal spectra, as used in this study. This is not as obvious as it might seem. Most article and textbook discussions omit these definitions and use the letter *S* for some spectral quantity with a variety of names like (1) power spectrum, (2) spectral power density, (3) power density spectrum, (4) energy density spectrum, (5) spectral density, (6) spectral intensity or (7) spectral noise power. Nevertheless, there is no commonly accepted definition whether the single sided variant (S_u) is meant or the double sided variant (D_u).

single (0…∞) *4kTR*	*double* (–∞…∞) *2kTR*	*reference*
-	2π·W	***Haus et.al.*** *"Representation of noise in linear twoports"* Proceedings of the IRE, jan 1960, p69-74
S	-	***Ziel*** "Noise: Sources, Characterization, Measurement", Prentice-Hall, Englewood Cliffs New Jersey 1970
-	S	***Hullett, Muoi*** *"A feedback receiver amplifier for Optical Transmission Systems"* IEEE Trans. on Communications, oct 1976, p 1180-1185
-	S	***Smith, Hooper, Garrett*** *"Receivers for optical comm: a comparison of avalanche photodiodes with PIN-FET hybrids"* Opt. and Quantum Elect. 10 (1978) p293-300
-	S	***Davidse*** Electronic amplifiers and phaselocked loops 1982. VSSD-Delft, the Netherlands (in Dutch).
S	-	***Nordholt*** "Design of high performance negative feedback amplifiers", Elseviers, Amsterdam, 1983, ISBN 0-444-42140-8
-	S	***Jain, Kumar, Gupta*** *"Design of an optimum optical receiver"* J. of Optical Communications, 6 (1985) 3, p106-112
S	-	***Gupta et.al.*** *"MW Noise characterization of GaAs MESFETS: Evaluation by ..."* IEEE transactions on MTT, vol 35, no 12 dec 1987, p1208-1217
-	S	***Howard, Jeffery, Hullett*** *"On the noise of high-transimpedance amplifiers for long-wavelength pulse OTDRs"* Optical and quantum electronics 19 (1987) p123-129
S	-	***Darcie, Kasper, Talman, Burrus*** *"Resonant pin-FET receivers for lightwave subcarier systems"* J. of Lichtwave technology vol 6, no 4, april 1988, p582-589
-	S	***Jacobsen, Kan, Garrett*** *"Improved design of tuned optical receivers"* Electronic letters, vol 23, no15, july 1987, p787-788
-	S	***Einarsson*** *"Error probability of optical receivers"* J. of Optical Communications, 11 (1990) 4, p137-141
S	-	***Schneider*** *"Reduction of spectral Noise Density in pin-HEMT lightwave receivers" J* Journal of lightwave Techn. vol 9, no 7, july 1991, p887-892
-	S	***Dobrowolski*** Introduction to computer methods for microwave circuit analysis and design. Artech House, Boston 1991, ISBN 0-89006-505-5

Fig 7.1 *A survey of various publications related to intensity spectra. The symbol* S *is often used, however, there is no commonly accepted convention whether the* single *or the* double *sided spectrum is meant. This illustrates the importance of defining this quantity to avoid confusion when noise spectra are discussed.*

To illustrate how confusing common practice is, figure 7.1 surveys various publications that use the letter *S* for intensity spectra, some of them indicating single sided spectra,

and others double sided spectra. Quite clearly, it is meaningless to specify noise levels *S* of amplifiers when an unambiguous definition of *S* is lacking. This is because *single* sided and *double* sided spectra differ by 3dB!
More confusion arises when one paragraph of a textbook specifies *S* as a double sided intensity spectrum while in another paragraph it must be concluded[3] from the thermal noise that *S* denotes a single sided spectrum.
Unambiguous definitions of *S* may sometimes be inferred based on physical quantities such as thermal noise[4] or ideal shot noise[5]. With other authors the relationship to the auto-correlation[6] function may give a clue. Many textbook discussions and publications use terms related to intensity spectra, that are employed in a rather loose fashion. It is the prime function of section 7.1 to provide the required definitions.

7.1.1. Spectral identification of random signals

Linear circuit analysis is relatively simple when using a spectral representation of the various signals. There is no unique definition of spectrum, since periodic and random signals have no Fourier transform due to the divergent Fourier integral when integrating from $-\infty$ to $+\infty$.
To ensure convergence of the Fourier integrals, signals must be switched on and off. This also holds when integrating the signal over a limited time interval, while assuming that the signal is zero outside this interval. Using this concept, the following spectra can be distinguished:

- For periodic signals, it is convenient to chose the length of the integration time interval equal to the period time of the signal. This yields the *discrete spectrum*, being the complex coefficients of the Fourier series expansion in complex form.
- For random signals, it is convenient to chose an interval T that is long enough to smooth the spectral rms-amplitude, averaged within a small frequency band $\Delta f > (1/T)$. This yields the *complex noise spectrum*.
- For a finite number of signal pulses, the total energy is finite. This means that integration from $-\infty$ to $+\infty$ yields convergent Fourier integrals and that the standard Fourier-transform is applicable. This provides the *continuous spectrum*.
- For auto-correlated random signals the total energy is also finite. There is an additional mathematical operation, to facilitate convergence of the Fourier integrals. This combined transformation yields the *intensity spectrum*, which is also called the power[7] spectrum or the power spectral density.

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[3] An example is Dobrowolski [724]. Equation 5.4 of [724] relates the power spectral density S(f) with the auto-correlation function, from which one concludes that it is a *double sided* intensity spectrum. From equation 5.7, 5.33 and 5.48 it is clear that the thermal noise current intensity of a resistor equals S(f)=4kT/R. This intensity is equal to the *single sided* intensity spectrum. This illustrates the confusion related to power spectra.

[4] Thermal noise is: $S_u(f)=4kTR$ and $D_u(f)=2kTR$ for voltages, or $S_i(f)=4kT/R$ and $D_i(f)=2kT/R$ for currents.

[5] Shot noise current, associated with dc-current I_{dc}, is: $S_i(f)=2q\cdot I_{dc}$ and $D_i(f)=q\cdot I_{dc}$

[6] The auto-correlation function is: $R_u(t)=\langle U(t)\cdot U^*(t+\tau)\rangle_\infty$

[7] We prefer the name *intensity* spectrum instead of *power* spectrum to avoid confusion with respect to the dimensions of the spectrum. Intensity spectra can be specified in A^2/Hz, in V^2/Hz or in W/Hz.

The mathematical definitions of all these spectra are summarized below. When there is no ground for confusion, the abbreviated notation will be used. Appendix *K* provides a compact summary of all related definitions and properties.

$$\mathbf{j}^{¥}_{d}\{U(t);T\} \overset{def}{=} \frac{1}{T}\cdot\int_{-½T}^{+½T} U(t)\cdot exp(-j\cdot 2\pi n\cdot t/T)\cdot dt \qquad Q_u(n) = \textit{discrete spectrum}$$

$$\mathbf{j}_{n}\{U(t);T\} \overset{def}{=} \frac{1}{\sqrt{½T}}\cdot\int_{-½T}^{+½T} U(t)\cdot exp(-j\cdot 2\pi f\cdot t)\cdot dt \qquad N_u(f) = \textit{complex noise spectrum}$$

$$\mathbf{j}_{c}\{U(t)\} \overset{def}{=} \int_{-\infty}^{+\infty} U(t)\cdot exp(-j\cdot 2\pi f\cdot t)\cdot dt \qquad F_u(f) = \textit{continuous spectrum}$$

$$\boldsymbol{D}\{U(t)\} \overset{def}{=} \int_{-\infty}^{+\infty} \langle U(t)\cdot U'(t+\tau)\rangle_{\infty}\cdot exp(-j\cdot 2\pi f\cdot \tau)\cdot d\tau \qquad D_u(f) = \textit{double sided intensity spectrum}$$

$$\boldsymbol{S}\{U(t)\} \overset{def}{=} 2\cdot\int_{-\infty}^{+\infty} \langle U(t)\cdot U'(t+\tau)\rangle_{\infty}\cdot exp(-j\cdot 2\pi f\cdot \tau)\cdot d\tau \qquad S_u(f) = \textit{single sided intensity spectrum}$$

In these formulae, U'(t) denotes the conjugate transpose[8] of the signal U(t). This operation is relevant when U(t) is the (complex) mathematical transformed form of some physical (real) signal. In addition, $\langle f(t)\rangle_{\Delta t}$ denotes the signal average over interval Δt of function f(t). See appendix *K* for an exact definition.
Note that the word *single* sided spectrum implies that negative frequencies are not used in the reverse transformation to the time domain, while the reverse transformation of the *double* sided spectrum uses positive and negative frequencies. It is a symmetrical function.

Noise spectra, averaged within a resolution bandwidth

The definition of complex noise spectra $N_u(f)$ enables simple relations between various signals and spectra. The most significant rules are embedded in the following well-known formulas:

$\langle N_u\cdot N'_u\rangle_{\Delta f}$	$\approx$	$S_u(f)$	*Wiener-Khintchine theorem*
$\int_0^{\infty} S_u(f)\cdot df$	$=$	$\langle \lvert U(t)\rvert^2\rangle_{\infty}$	*Parceval theorem*

The Wiener-Khintchine theorem[9] [703,704] holds for sufficiently small resolution bandwidth Δf, provided that the selected interval length T is sufficiently large (T>1/Δf). For a proof of this theorem, see van der Ziel [710,720].
The Parceval theorem illustrates why S_u is called a *single sided* spectrum. The integral is restricted to positive frequencies. The same result would have been obtained with the *double sided* spectrum D_u when 'negative' frequencies are included for integration from (–∞…+∞).

Spectrum analyzer measurements

Spectrum analyzers measure none of these spectra directly. These selective voltmeters, with top detector and resolution bandwidth Δf, indicate voltages similar to:

when detecting harmonic[10] signals: → indicating: $\approx |Q_u(f)|$

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8 Note that U(t) is in this situation a scalar quantity, not a vector or matrix of scalar quantities. For scalar quanties holds that $U'(t) \equiv U^*(t)$, in which $U^*(t)$ represents the complex conjugate of U(t). For matrix quantities holds $\mathbf{U}'(t) \equiv \mathbf{U}^{*T}(t)$. The use of a conjugated *transpose* in our definition simplifies a generalization of this definition.

9 We note that the original theorem states that $S_u \rightarrow 2\cdot\int_{-\infty}^{+\infty}\langle U(t)\cdot U'(t+\tau)\rangle_{\infty}\cdot exp(-j\cdot 2\pi f\cdot\tau)\cdot d\tau$ and $S_u \overset{def}{=} \langle N_u\cdot N'_u\rangle_{\Delta f}$

10 Harmonic signals originate from the composition of a finite number of periodical signals.

when detecting noisy signals: $\rightarrow$ indicating: $\approx |N_u(f)|\sqrt{\Delta f} \rightarrow \sqrt{S_u \cdot \Delta f}$

Modern spectrum analyzers, with built-in noise markers, automatically modify the readout to correct for the resolution bandwidth of the instrument. The voltage at the noise marker frequency approximates $|N_u(f)|$. The video filter averages the detected $|N_u(f)|$ over the resolution bandwidth.

The smaller the video bandwidth the more the noise marker indication approaches $\sqrt{S_u}$ when noisy signals are detected. The Wiener-Khintchine theorem predicts that replacement of the top-detector by a true-rms detector improves the accuracy of this result.

7.1.2. Spectral correlation between random signals

Consider noisy voltages $V_p(t)$, in which p denotes an index ranging from 1 to n. The associated complex noise spectra[11] are represented as $V_p(f)$. In most practical situations, *complex* noise spectra are not available from measurements. This is because the spectral magnitude and phase are rapidly fluctuating. More appropriate spectral quantities are intensity spectra since they represent averaged quantities.

According to the Wiener-Khintchine theorem the intensity spectrum is a good measure for the rms-magnitude of complex noise spectra, averaged in a small frequency band Δf. The *cross-correlation intensity spectrum* (cross-spectral intensity) is a similar quantity, and is a suitable measure for the average phase relation between two signals. When analyzing noisy circuits with many noise sources, their (cross) intensity spectra are most conveniently organized in matrix form. These spectra are defined[12] as:

$$\mathbf{C} \overset{\text{def}}{=} \langle \mathbf{V} \cdot \mathbf{V}' \rangle_{\Delta f} = \begin{bmatrix} S_1 & S_{1|2} & \cdots & S_{1|n} \\ S_{2|1} & S_2 & \cdots & S_{2|n} \\ \cdots & \cdots & \cdots & \cdots \\ S_{n|1} & S_{n|2} & \cdots & S_n \end{bmatrix}$$

$$S_{p|q} \overset{\text{def}}{=} 2 \cdot \int_{-\infty}^{+\infty} \langle V_p(t) \cdot V_q'(t+\tau) \rangle_\infty \cdot exp(-j \cdot 2\pi f \cdot \tau) \cdot d\tau$$

$$\gamma_{pq} \overset{\text{def}}{=} \gamma'_{qp} \overset{\text{def}}{=} S_{p|q} / \sqrt{S_p \cdot S_q} = \frac{\langle V_p \cdot V'_q \rangle}{\sqrt{\langle |V_p|^2 \rangle \cdot \langle |V_q|^2 \rangle}}$$

$$C_{p|q} = S_{p|q} = \gamma_{pq} \cdot \sqrt{S_p} \cdot \sqrt{S_q} = \langle V_p(f) \cdot V_q'(f) \rangle_{\Delta f}$$

$\mathbf{V}(f)'$ *is the conjugate transpose of* $\mathbf{V}(f)$
$\mathbf{C}$ = *noise correlation matrix* = $\mathbf{C}'$

$S_{p|q} = C_{p|q}$ = *cross-correlation intensity spectrum*
$S_p = S_{p|p}$ = *(self) intensity spectrum*
$\gamma_{p|q}$ = *normalized correlation coefficient*

The normalized correlation coefficients $\gamma_{p|q}$ are dimensionless, and the magnitudes of all these (complex) coefficients are bounded by [0…1]. These coefficients can also be organized in matrix format, termed as the *normalized noise correlation matrix* γ.

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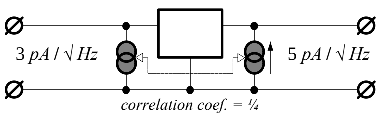
[11] In this case we use a shorthand notation, given by: $V(f) \overset{\text{def}}{=} N_V(f) \overset{\text{def}}{=} \mathbf{j}_n\{V(t);T\}$.

[12] The interpretation of these matrix definitions can be clarified using a model of a noisy amplifier. The model illustrated below shows two correlated noise sources, one at the amplifier input and the other at the output. The correlation matrix represents the spectral intensities of these sources in a well-organized way.



$S_1 = (3\,pA/\sqrt{Hz})^2$
$S_2 = (5\,pA/\sqrt{Hz})^2$
$\gamma = ¼$

$$\mathbf{C} = \begin{bmatrix} 3^2 & ¼ \cdot 3 \cdot 5 \\ ¼ \cdot 3 \cdot 5 & 5^2 \end{bmatrix} [pA/\sqrt{Hz}]^2$$

The Wiener-Khintchine theorem in matrix notation $\mathbf{C} = \langle \mathbf{V} \cdot \mathbf{V'} \rangle_{\Delta f}$ provides the simple means to relate the measurable matrix quantity **C** to the theoretical vector quantity **V**. This is an important property when applying circuit analysis to noisy circuits.
Other names for noise correlation matrix **C** that have been used from the beginning are: coherence matrix [Wiener: 709], noise spectral matrix [Haus and Adler: 716], power spectrum matrix [Gamo: 717], or the noise power matrix [Bosma: 719]. More recent publications tend to use the term *correlation matrix*, e.g. [Hillbrandt and Russer: 721].

7.1.3. Analytical noise analysis using complex noise spectra

Noise signals are usually quantified by spectral intensity and their correlation with other noisy signals. This is because rapid fluctuations of noisy signals interfere with reliable measurement of the phase of their complex noise spectra. It is, therefore, common practice to base noise analysis on intensity spectra.
It is unhandy when (manual) analyzing noisy circuits to use intensity spectra at each step of the analysis. But assuming that the magnitude and phase of complex noise spectra are *completely known*, all spectral analysis methods for deterministic signals can be applied to noisy signals. It is more convenient to ignore the lack of phase information, and act as if noisy signals are deterministic. Using this approach, the intensity spectrum of a noisy signal of interest is evaluated as follows:

- Identify each noise source with a unique symbol, such as V(f), J(f) or W(f), representing the complex noise spectrum of that source.
- Derive an analytical expression for the output noise (or any other equivalent noise source of interest) using standard AC-analysis techniques.
- Transform this analytical expression for the complex noise spectrum into an expression for the intensity spectrum.

For instance, let $N(f)=\alpha\cdot V+\beta\cdot J$ be an expression, derived for the complex noise spectrum of interest. In this expression denote V(f) and J(f) the complex noise spectra of a voltage- and current noise source respectively. The circuit dependent variables $\alpha(f)$ and $\beta(f)$ are expressed in terms of resistance, capacitance, etc. The transformation of a complex noise spectrum N(f) into its associated intensity spectrum S(f) is:

$$S = \langle N\cdot N' \rangle = \langle (\alpha\cdot V+\beta\cdot J)\cdot(\alpha'\cdot V'+\beta'\cdot J') \rangle$$
$$S = \langle \alpha\cdot V\cdot\alpha'\cdot V' \rangle + \langle \beta\cdot J\cdot\beta'\cdot J' \rangle + \langle \alpha\cdot V\cdot\beta'\cdot J' \rangle + \langle \beta\cdot J\cdot\alpha'\cdot V' \rangle$$
$$S = \alpha\cdot\alpha'\cdot\langle V\cdot V' \rangle + \beta\cdot\beta'\cdot\langle J\cdot J' \rangle + \alpha\cdot\beta'\cdot\langle V\cdot J' \rangle + \alpha'\cdot\beta\cdot\langle J\cdot V' \rangle$$
$$S = \alpha\cdot\alpha'\cdot S_V + \beta\cdot\beta'\cdot S_J + \alpha\cdot\beta'\cdot S_{V|J} + \alpha'\cdot\beta\cdot S_{J|V}$$
$$S = |\alpha|^2\cdot S_V + |\beta|^2\cdot S_J + 2\cdot re\{\alpha\cdot\beta'\cdot S_{V|J}\}$$

When numerical values for S_V, S_J, $S_{V|J}$ and $S_{J|V}$ have been experimentally determined, the numerical value for S can be evaluated.

This analytical approach using complex noise spectra instead of intensity spectra is convenient for *manual* noise analysis of circuits. On the other hand, most circuit simulators provide numerical results instead of analytical expressions. The analytical approach is thus inconvenient for use with circuit simulators. Section 7.2.4 describes an alternative method using noise correlation matrices that is more suited to *automated* noise analysis.

7.2. Blackbox representation of noisy multi-port networks

The validation of equivalent circuit models is limited when modeling noisy linear devices with lumped noise sources. The more parasitic effects are taken into account, the wider the frequency interval will be in which the model is applicable.
Wideband analysis of noisy circuits requires a more robust approach. Blackbox representations of linear noisy circuits separate the physical origin of internal noise sources from what is externally perceptible. Blackbox methods are more widely applicable and provide a good starting-point for developing adequate device noise models.
Section 7.2 extends the blackbox analysis of linear circuits, as described in section 2.2, to include circuits with internal sources. It starts from the basic analysis of circuits comprising deterministic signal sources to construct a systematic analysis of arbitrary noisy circuits.

The voltage, current and wave methods described here are in essence known from the literature. In practice, publications focus on one or two of these three, confining the intercomparison of the methods. Furthermore, the wave methods discussed in the literature are often restricted to real reference impedances. One aim of this chapter is to discuss all the methods *simultaneously*, and to extend the application of the wave methods to ports normalized to arbitrary (complex) reference impedances.

7.2.1. Description of multi-port circuits including deterministic sources

Consider a multinode circuit for which all nodes are externally accessible via ports. A Norton representation of an internal signal source results in a current source. This current flow between two nodes, for instance directed from node n_1 to n_2, is equivalent to re-routing the current flow via an arbitrary third node, e.g. the ground node. Therefore, a current source between two nodes is equivalent to two identical current sources: one directed from node n_1 to ground and another directed from ground to node n_2. This is illustrated in figure 7.2.

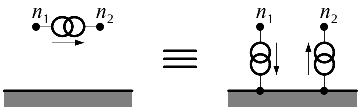
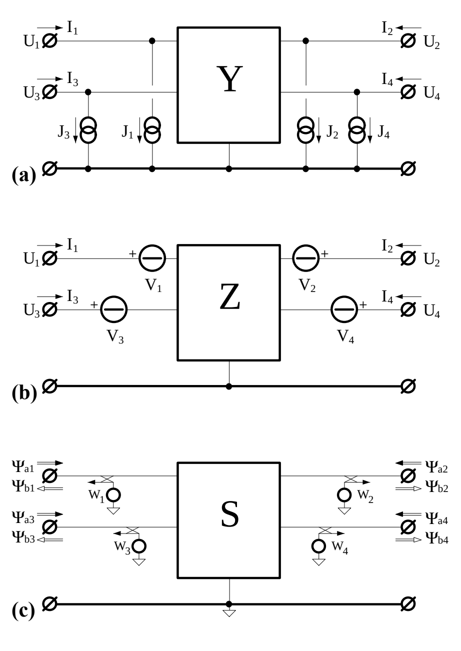


Fig 7.2 *A current source connected between nodes is equivalent to two identical current sources.*

When the circuit comprises two or more sources, this approach holds for any individual source. Figure 7.3a shows the combined equivalent circuit model, being an extended Norton model. Each equivalent current source between a port node and ground originates from the summation of all current sources connected with that node and re-routed via ground.
This extended Norton model of multi-ports links up nicely when describing the multi-port with y-parameters. Figure 7.3a also shows the associated admittance matrix equation. Reorganization of these matrix equations results in the reverse situation, as is shown in figure 7.3b. It provides an extended Thévenin model, in which all current

sources are transformed into voltage sources. This model links up nicely with z-parameters. A similar approach holds for waves, as is shown in figure 7.3c.



Norton representation

$$\begin{bmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ \dots & \dots & \dots & \dots \\ y_{n1} & y_{n2} & \dots & y_{nn} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ \dots \\ U_n \end{bmatrix} + \begin{bmatrix} J_1 \\ J_2 \\ \dots \\ J_n \end{bmatrix}$$

$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{U} + \mathbf{J}$$

Thévenin representation

$$\begin{bmatrix} U_1 \\ U_2 \\ \dots \\ U_n \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1n} \\ z_{21} & z_{22} & \dots & z_{2n} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nn} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ \dots \\ V_n \end{bmatrix}$$

$$\mathbf{U} = \mathbf{Z} \cdot \mathbf{I} + \mathbf{V}$$

Wave representation

$$\begin{bmatrix} \Psi_{b1} \\ \Psi_{b2} \\ \dots \\ \Psi_{bn} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & s_{22} & \dots & s_{2n} \\ \dots & \dots & \dots & \dots \\ s_{n1} & s_{n2} & \dots & s_{nn} \end{bmatrix} \cdot \begin{bmatrix} \Psi_{a1} \\ \Psi_{a2} \\ \dots \\ \Psi_{an} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \dots \\ W_n \end{bmatrix}$$

$$\mathbf{\Psi}_b = \mathbf{S} \cdot \mathbf{\Psi}_a + \mathbf{W}$$

Fig 7.3 *Three basic blackbox models of multi-port networks comprising arbitrary (deterministic) signal sources. They are a generalization of the Norton, Thévenin and wave representations of one-port sources, as described in section 2.1.3. The exchangeable source power is [715]:*

$$P_e = \tfrac{1}{2} \cdot \mathbf{V'}/(\mathbf{Z}+\mathbf{Z'}) \cdot \mathbf{V} = \tfrac{1}{2} \cdot \mathbf{J'}/(\mathbf{Y}+\mathbf{Y'}) \cdot \mathbf{J} = \mathbf{W'}/(\sigma - \mathbf{S} \cdot \sigma \cdot \mathbf{S'}) \cdot \mathbf{W}$$

In section 2.2.1, transformation rules were summarized between y-, z- and s-parameters. The s-parameter transformations apply to waves normalized to arbitrary reference impedances Z_N (port normalization numbers). The reference impedances for the individual ports are applicable, including complex impedances with negative real part.
Additional transformation rules for the equivalent sources J, V and W are summarized in figure 7.4. They are simply reducible from the relations between voltages, currents and waves, as defined in section 2.1.1.
All transformation rules for sources have in common that the transformation reduces to a simple matrix product. This is an important property when extracting statistical properties of these (noise) sources.

	Y		Z		S
$\mathbf{J} = \Phi_{YZ} \cdot \mathbf{V}$	$\Phi_{YZ} = -\mathbf{Y}$	=	$-\mathrm{inv}(\mathbf{Z})$	=	$\mathbf{r}_N \cdot ((\mathbf{S} \cdot \mathbf{Z}_N + \mathbf{Z}_N')\backslash(\mathbf{1}\ \ \mathbf{S}))/\mathbf{r}_N$
$\mathbf{V} = \Phi_{ZY} \cdot \mathbf{J}$	$\Phi_{ZY} = -\mathrm{inv}(\mathbf{Y})$	=	$-\mathbf{Z}$	=	$\mathbf{r}_N \cdot ((\mathbf{1}\ \ \mathbf{S})\backslash(\mathbf{S} \cdot \mathbf{Z}_N + \mathbf{Z}_N'))/\mathbf{r}_N$
$\mathbf{J} = \Phi_{YS} \cdot \mathbf{W}$	$\Phi_{YS} = -(\mathbf{Y} \cdot \mathbf{Z}_N + \mathbf{1})/(\sigma \cdot \mathbf{r}_N)$	=	$-(\mathbf{Z}\backslash\mathbf{Z}_N + \mathbf{1})/(\sigma \cdot \mathbf{r}_N)$	=	$-(2 \cdot \mathbf{r}_N)/(\mathbf{S} \cdot \mathbf{Z}_N + \mathbf{Z}'_N)$
$\mathbf{W} = \Phi_{SY} \cdot \mathbf{J}$	$\Phi_{SY} = -(\sigma \cdot \mathbf{r}_N)/(\mathbf{Y} \cdot \mathbf{Z}_N + \mathbf{1})$	=	$-(\sigma \cdot \mathbf{r}_N)/(\mathbf{Z}\backslash\mathbf{Z}_N + \mathbf{1})$	=	$-(\mathbf{S} \cdot \mathbf{Z}_N + \mathbf{Z}'_N)/(2 \cdot \mathbf{r}_N)$
$\mathbf{V} = \Phi_{ZS} \cdot \mathbf{W}$	$\Phi_{ZS} = (\mathbf{Z}_N + \mathbf{Y}\backslash\mathbf{1})/(\sigma \cdot \mathbf{r}_N)$	=	$(\mathbf{Z}_N + \mathbf{Z})/(\sigma \cdot \mathbf{r}_N)$	=	$(2 \cdot \mathbf{r}_N)/(\mathbf{1} - \mathbf{S})$
$\mathbf{W} = \Phi_{SZ} \cdot \mathbf{V}$	$\Phi_{SZ} = (\sigma \cdot \mathbf{r}_N)/(\mathbf{Z}_N + \mathbf{Y}\backslash\mathbf{1})$	=	$(\sigma \cdot \mathbf{r}_N)/(\mathbf{Z}_N + \mathbf{Z})$	=	$(\mathbf{1} - \mathbf{S})/(2 \cdot \mathbf{r}_N)$

Fig 7.4 *Transformation rules for the equivalent sources of figure 7.3. All of these transformations share the form:* $\mathbf{X}_b = \Phi_{ba} \cdot \mathbf{X}_a$. *For the definition of* σ, *and* $\mathbf{r}_N$, *see section 2.2.1. The transformation matrices using s-parameters hold for waves normalized to arbitrary reference impedances* Z_n.

7.2.2. Description of noisy multi-ports using correlation matrices

There are many blackbox descriptions for describing noisy multi-ports. More convenient methods have in common that noise and transfer descriptions are closely matched. When matrix descriptions are chosen, standard blackbox models are preferred, similar to figure 7.3. This requires the replacement of deterministic sources by noise sources, and the representation of these equivalent sources by complex noise spectra J, V or W.

Complex noise spectra are primarily intended for use as intermediate results and are not suitable for quantify noise sources. Self- and cross-correlation intensity spectra are more appropriate, and can be organized in correlation matrix format. For Y-, Z- and S-matrix transfer descriptions we define[13] the associated noise correlation matrices as follows:

$$\mathbf{C}_Y \stackrel{\text{def}}{=} \langle \mathbf{J} \cdot \mathbf{J}' \rangle_{\Delta f} \qquad = \text{noise current correlation matrix}$$

$$\mathbf{C}_Z \stackrel{\text{def}}{=} \langle \mathbf{V} \cdot \mathbf{V}' \rangle_{\Delta f} \qquad = \text{noise voltage correlation matrix}$$

$$\mathbf{C}_S \stackrel{\text{def}}{=} \langle \mathbf{W} \cdot \mathbf{W}' \rangle_{\Delta f} \qquad = \text{noise wave correlation matrix}$$

Let's consider an arbitrary circuit topology, with various uncorrelated noise sources. Common circuit theory is applicable when the individual noise sources are represented by complex noise spectra. Then, perform the same calculation as with deterministic sources. This to extract the desired transfer matrix of the blackbox representation, and its associated equivalent noise sources. Finally, convert[14] their complex noise spectra into the desired correlation matrix format. This approach is much less complicated for manual analysis than using correlated intensity spectra at each stage of the analysis.

---

[13] Some authors use definitions that are 'normalized' with respect to thermal noise: all matrices are divided by kT.

[14] For instance, let J_1 and J_2 be equivalent noise sources of a Norton representation of a circuit. They are linearly related to some uncorrelated internal sources J_a and J_b with intensity spectra S_a and S_b. The element value $\langle J_1 \cdot J'_2 \rangle$ of the admittance noise correlation matrix $\mathbf{C}_Y$ is extracted according to:

$$\left.\begin{array}{l} J_1 = \alpha_1 \cdot .J_a + \alpha_2 \cdot .J_b \\ J_2 = \alpha_3 \cdot .J_a + \alpha_4 \cdot .J_b \end{array}\right\} \Rightarrow \begin{array}{ll} J_1 \cdot J'_2 & = \alpha_1 \cdot \alpha'_3 \cdot J_a \cdot J'_a + \alpha_2 \cdot \alpha'_4 \cdot J_b \cdot J'_b + \alpha_1 \cdot \alpha'_4 \cdot J_a \cdot J'_b + \alpha_2 \cdot \alpha'_3 \cdot J_b \cdot J'_a \\ \langle J_1 \cdot J'_2 \rangle & = \alpha_1 \cdot \alpha'_3 \cdot S_a + \alpha_2 \cdot \alpha'_4 \cdot S_b + 0 + 0_a \end{array}$$

Using noise correlation matrices associated with y-, z- or s-parameters has many advantages. These advantages are decisive to simplify automated noise analysis on arbitrary circuits.
At first, the composition of $\mathbf{C}_Y$ for arbitrary circuits can be performed in a plain way. In addition, it is easy to convert $\mathbf{C}_Y$ into $\mathbf{C}_Z$ or $\mathbf{C}_S$ using simple matrix products. Furthermore, when most nodes of a circuit are not externally available as port nodes, the reduction of the correlation matrix dimension is performed similar to that of transfer matrices. The are elucidated below.

Composition rule for $\mathbf{C}_Y$
Matrix $\mathbf{C}_Y$ of a complex noisy circuit originates from the matrix addition of correlation matrices of all its individual devices. This is because (1) shunted current sources will add and (2) the noise of one device is uncorrelated with the noise of another one. As a result:

$$\boxed{\mathbf{C}_{Y,\mathrm{circuit}} = \sum \mathbf{c}_{Y,\mathrm{devices}}}$$

This composition rule is restricted to admittance noise correlation matrices only. Its application in circuit simulators will be discussed in section 7.2.4.

Transformation rules for correlation matrices
The transformation rules of figure 7.4 for equivalent currents to equivalent voltages or waves are also applicable to complex noise spectra. Let $\mathbf{X}_a$ be a vector representing equivalent sources to be transformed into vector $\mathbf{X}_b$. The transformation of an arbitrary $\mathbf{X}_a$ into $\mathbf{X}_b$ equals: $\mathbf{X}_b=\mathbf{\Phi}_{ba}\cdot\mathbf{X}_a$, in which matrix $\mathbf{\Phi}$ is the associated transformation matrix.
This formula provides the transformation rule of an arbitrary noise correlation matrix $\mathbf{C}_a$ to $\mathbf{C}_b$ and has the following general form:

$$\mathbf{C}_b = \langle \mathbf{X}_b\cdot\mathbf{X'}_b \rangle = \langle (\mathbf{\Phi}_{ba}\cdot\mathbf{X}_b)\cdot(\mathbf{\Phi}_{ba}\cdot\mathbf{X}_b)' \rangle = \mathbf{\Phi}_{ba}\cdot\langle \mathbf{X}_b\cdot\mathbf{X'}_b \rangle\cdot\mathbf{\Phi'}_{ba}$$

$$\boxed{\mathbf{C}_b = \mathbf{\Phi}_{ba}\cdot\mathbf{C}_a\cdot\mathbf{\Phi'}_{ba}}$$

The general transformation matrices $\mathbf{\Phi}$ in figure 7.4 are all applicable to transforming noise correlation matrices. The most important relations are:

$$\mathbf{C}_Z = \mathbf{Z}\cdot\mathbf{C}_Y\cdot\mathbf{Z'}$$
$$\mathbf{C}_Y = \mathbf{Y}\cdot\mathbf{C}_Z\cdot\mathbf{Y'}$$
$$\mathbf{C}_S = \mathbf{Q}\cdot\mathbf{C}_Z\cdot\mathbf{Q'} \qquad \mathbf{Q}=(\mathbf{1}-\mathbf{S})/(2\cdot\mathbf{r}_N)$$

Correlation matrices have been known for years. In addition, Haus and Adler [715,716] derived in 1958 the natural regularity of the above transformation, as intermediate result in discussing properties for noise measure. Bosma [719] derived in 1967 the same regularity, when treating these transformations as mathematical coordinate transformations in a signal-state space. Just in 1976, Hillbrand and Russer [721] illustrated the importance of correlation matrices for the analysis of noisy circuits. They showed how to apply these matrices for simplifying two-port noise analysis.

7.2.3. Reduction of correlation matrix dimension of noisy multi-ports

Consider a circuit with N_n nodes, represented by an $N_n \times N_n$ matrix (y-, z- or s-parameters). In many circuit applications, no more then a few nodes ($N_p < N_n$) are externally available, for instance nodes p_1 and p_2 (the port nodes). This means that all Thévenin voltage sources of figure 7.3b are floating on one side, except those connected to the port nodes. As a result, these voltage sources are unused and can be omitted.
In section 2.2.3 a similar situation was discussed. It explained that the matrix reduction of the *Thévenin* representation is simply the deletion of all rows and columns associated with internal nodes.
Let **U** = **Z**·**I** + **V** be the original matrix representation of a multi-port network, and let **u** = **z**·**i** + **v** be its reduced matrix representation. The reduced signal vectors **u**, **i** and **v** originate from **U**, **I** and **V** by deletion of all rows, except those with number p_1 and p_2 The same applies for the rows and columns of **Z** to generate **z**. Using the port definition matrix **P** (see section 2.2.3) for simplifying this reduction formalism yields:

$$\mathbf{P} \stackrel{\text{def}}{=} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} \leftarrow \text{row p1} \\ - \\ \leftarrow \text{row p2} \\ - \\ - \\ - \end{matrix} \quad \Rightarrow \quad \begin{cases} \mathbf{i} = \mathbf{P'}\cdot\mathbf{I} \\ \mathbf{u} = \mathbf{P'}\cdot\mathbf{U} \\ \mathbf{v} = \mathbf{P'}\cdot\mathbf{V} \\ \mathbf{z} = \mathbf{P'}\cdot\mathbf{Z}\cdot\mathbf{P} \end{cases}$$

This matrix identity is reserved for the Thévenin representation only (z-parameters). Reduction of y- and s-parameters requires z-parameters as intermediate representation format. Use the transformation rules of figure 7.4 for this purpose, for instance: $\mathbf{y} = inv(\mathbf{P'}/\mathbf{Y}\cdot\mathbf{P})$ and $\mathbf{i} = (\mathbf{y}\cdot\mathbf{P'})\cdot(\mathbf{Y}\backslash\mathbf{I})$.

A similar reduction technique can be performed with correlation matrices. Consider a circuit with N_n nodes, of which the voltage noise correlation matrix is known. Let $\mathbf{C}_Z$ be a correlation matrix and **V** be a noisy voltage source vector to be reduced to matrix $\mathbf{c}_Z$ and vector **v**. Using the above port definition matrix **P**, the reduction of the source vector is performed with the multiplication **v**=**P'**·**V**. The reduced correlation matrix $\mathbf{c}_Z$ is:

$$\mathbf{c}_Z = \langle \mathbf{v}\cdot\mathbf{v'} \rangle = \langle (\mathbf{P'}\cdot\mathbf{V})\cdot(\mathbf{P'}\cdot\mathbf{V})' \rangle = \mathbf{P'}\cdot\langle \mathbf{V}\cdot\mathbf{V'} \rangle\cdot\mathbf{P}$$

$$\boxed{\mathbf{c}_Z = \mathbf{P'}\cdot\mathbf{C}_Z\cdot\mathbf{P}}$$

This reduction rule is restricted to voltage noise correlation matrices only. Reduction of $\mathbf{C}_Y$ and $\mathbf{C}_S$ requires $\mathbf{C}_Z$-parameters as intermediate representation format. Use the transformation matrices of figure 7.4 for this purpose.
All these reduction techniques are important when implementing circuit simulators as described in section 2.2.4 and section 7.2.4.

7.2.4. Application of noise correlation matrices in circuit simulators.

Circuit simulators can easily be extended with noise analysis facilities when using y-parameters for transfer analysis and $\mathbf{C}_Y$ correlation matrices for noise analysis. This method has been demonstrated in 1985 by Rizzoli and Lipparini [723]. Because our integrated approach of designing and characterizing wideband circuits relies extensively on various computer algorithms, this section discusses algorithms for automated analysis of noisy circuits in more detail.

Consider an arbitrary circuit topology, of which all elements are subsequently described in a netlist. The admittance matrix **Y** of this circuit is constructed as discussed in section 2.2.4. The associated correlation matrix $\mathbf{C}_Y$ is constructed simultaneously by insertion of each individual noise source, as described below:

- An uncorrelated current noise source from node n_1 to n_2 with spectral intensity S_i is modeled by two fully correlated sources. One current flowing from node n_1 to ground, and another flowing with opposite sign from node n_2 to ground. This has been illustrated in figure 7.2. The associated device correlation matrix $\mathbf{C}_Y$ is as follows added to the circuit correlation matrix $\mathbf{C}_Y$:

$$\begin{bmatrix} c_{n2,n1} & c_{n2,n3} \\ c_{n4,n1} & c_{n4,n3} \end{bmatrix} = \begin{bmatrix} c_{n2,n1} & c_{n2,n3} \\ c_{n4,n1} & c_{n4,n3} \end{bmatrix} + S_i \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- Two or more correlated current noise sources are inserted similarly to a single uncorrelated source. The difference is that the device correlation matrix $\mathbf{c}_Y$ is a 4×4 matrix. It represents the (cross) intensity spectra of four equivalent sources from the four nodes to ground.

  Let S_a, S_b, S_{ab} and S_{ba} be the associated (cross) intensity spectra ($S_{ba}=S'_{ab}$). A simple way to expand the 4×4 device correlation matrix is provided by the matrix product:

$$\mathbf{c}_Y = \begin{bmatrix} +1 & -1 & 0 & 0 \\ 0 & 0 & +1 & -1 \end{bmatrix}' \cdot \begin{bmatrix} S_a & S_{ab} \\ S_{ba} & S_b \end{bmatrix} \cdot \begin{bmatrix} +1 & -1 & 0 & 0 \\ 0 & 0 & +1 & -1 \end{bmatrix} \qquad (\mathbf{Z}' \text{ is the transpose of } \mathbf{Z})$$

- An uncorrelated voltage noise source, in series with a non-zero impedance, is inserted in $\mathbf{C}_Y$ by using its Norton current representation. When this series impedance is zero, a non-zero impedance is imitated with two resistors: +R and –R. The positive resistor is subsequently used for converting voltage sources into current sources. The consequence of this approach is allocating additional (internal) nodes.
- Thermal noise of resistors R can be inserted like uncorrelated noise currents. Each resistor is shunted with a noise current with intensity spectrum $S_i=4kT/R$. The same applies for shot noise sources with intensity $S_i=2q\cdot I_{dc}$ due to dc-currents I_{dc}.

  Insertion of thermal noise of a passive reciprocal sub-circuit in thermal equilibrium, such as a lossy transmission line, can be performed by evaluating $\mathbf{c}_Y$ of the sub-circuit and addition of $\mathbf{c}_Y$ to $\mathbf{C}_Y$. Section 7.2.5 discusses how to evaluate $\mathbf{c}_Y$ from the transfer matrix **Y**.

When all noise sources of the net-list are inserted, the combined correlation matrix $\mathbf{C}_Y$ is complete. The matrix $\mathbf{C}_Z=\mathbf{Z}\cdot\mathbf{C}_Y\cdot\mathbf{Z}'$ provides the associated Thévenin representation of the noisy circuit. The circuit simulator must now reduce this matrix into a smaller correlation matrix, for instance a 2×2 matrix $\mathbf{c}_Z$ of a two-port. As described in section 7.2.3 the reduced correlation matrix $\mathbf{c}_Z$ equals:

$$\mathbf{c}_Z = \mathbf{P}' \cdot \mathbf{C}_Z \cdot \mathbf{P} = \mathbf{P}' \cdot (\mathbf{Z} \cdot \mathbf{C}_Y \cdot \mathbf{Z}') \cdot \mathbf{P}$$

$$\boxed{\mathbf{c}_Z = (\mathbf{P}'\cdot\mathbf{Z})\cdot\mathbf{C}_Y\cdot(\mathbf{P}'\cdot\mathbf{Z})'}$$

The circuit simulator has completed when all $\mathbf{c}_Z$ matrices are evaluated for all frequencies of interest. When $\mathbf{c}_Y$- or $\mathbf{c}_S$-matrices are preferred, convert all $\mathbf{c}_Z$ matrices into the preferred format. The dimension of matrix $\mathbf{c}_Z$ depends on the number of ports of the circuit, and may differ from two-ports.
We extended the circuit simulator of section 2.2.4 with noise analysis facilities in the 4GL computer language MatLab© [123], and found that less then one or two additional pages source code are adequate for implementing the overall simulator.

7.2.5. Generalized thermal noise theorem for multi-port networks

In a practical situation, there is no need for measuring the thermal noise of a resistor. A simple resistance and temperature measurement is adequate for predicting this noise using the well-known Nyquist theorem [702]: $S_u=4\cdot kT\cdot R$. Complete proofs of this theorem can also be found in van der Ziel [710,720].
A generalized theorem facilitates the thermal noise calculation for complex impedances Z. The thermal noise voltage can be calculated from impedance measurements using $S_u=4\cdot kT\cdot re\{Z\}$.
This simple thermal noise theorem can further be generalized for arbitrary passive multi-ports in thermal equilibrium, e.g. two-port filters with lossy inductors and distributed (lossy) elements. It facilitates the evaluation of all three basic equivalent noise models shown in figure 7.3, merely from transfer measurements. Noise measurements are not required.
The thermal noise correlation matrices, associated with the equivalent noise models of figure 7.3, of passive reciprocal multi-ports in thermal-equilibrium are:

$$\begin{array}{lll} \mathbf{C}_Y = \langle \mathbf{J}\cdot\mathbf{J}'\rangle_{\Delta f} & = 2kT\cdot(\mathbf{Y}+\mathbf{Y}') & \rightarrow S_J = 4kT\cdot re\{Y\} \\ \mathbf{C}_Z = \langle \mathbf{V}\cdot\mathbf{V}'\rangle_{\Delta f} & = 2kT\cdot(\mathbf{Z}+\mathbf{Z}') & \rightarrow S_V = 4kT\cdot re\{Z\} \\ \mathbf{C}_S = \langle \mathbf{W}\cdot\mathbf{W}'\rangle_{\Delta f} & = kT\cdot(\sigma-\mathbf{S}\cdot\sigma\cdot\mathbf{S}') & \rightarrow S_W = kT(1-|\Gamma|^2) \end{array}$$

k = Boltzmann constant, $1.380662\cdot10^{-23}$[J/K]
T = absolute temperature, in [K]
for definition of **σ** see section 2.2.1

spectral intensity of one-port device

Proof
The calculation methods of section 7.2.4 for constructing correlation matrices are the bases for proofing the above generalized thermal noise theorems.
When constructing the admittance transfer matrix **Y** and the associated current correlation matrix $\mathbf{C}_Y$ of arbitrary passive noisy circuits, one can apply the methods that are discussed in section 2.2.4 and 7.2.4 for circuit simulators. The relation between $\mathbf{C}_Y$ and **Y** for arbitrary passive circuits is derived in the following steps:

(a) The thermal noise contribution from individual resistors R equals: $S_i=4\cdot kT/R$. The way their admittance (1/R) is inserted in matrix **Y** equals the way their noise S_i is inserted in matrix $\mathbf{C}_Y$. As a result, $\mathbf{C}_Y=4\cdot kT\cdot\mathbf{Y}$ when all circuit elements are resistors at equal temperature.

(b) Inductors and capacitors do not contribute to the generation of thermal noise, and their admittance is pure imaginary. As a result, $\mathbf{C}_Y=4\cdot kT\cdot re\{\mathbf{Y}\}$ when all circuit elements are resistors, capacitors and inductors at equal temperature.
(c) These passive networks are all reciprocal networks, see section 2.2.2, which means that $\mathbf{Y}'=conj\{\mathbf{Y}^T\}=conj\{\mathbf{Y}\}$ and that $re\{\mathbf{Y}\}=(\mathbf{Y}+\mathbf{Y}')$. As a result, $\mathbf{C}_Y=2\cdot kT\cdot(\mathbf{Y}+\mathbf{Y}')$ for passive circuits in thermal equilibrium.

Using the transformation rules for the matrices $\{\mathbf{Y}, \mathbf{C}_Y\}$ to $\{\mathbf{Z}, \mathbf{C}_Z\}$ or $\{\mathbf{S}, \mathbf{C}_S\}$, as discussed in section 2.2.1 and 7.2.2, provides the associated correlation matrices.

Validity
These expressions also hold for networks of which no more then a few nodes are externally available (port nodes). This can easily be verified for $\mathbf{C}_Z$ since the reduction formula for matrix $\mathbf{C}_Z$ (see section 7.2.3) equals the reduction formula for matrix $\mathbf{Z}$ (see section 2.2.3).
To the author knowledge, all publications evaluating $\mathbf{C}_S$ for thermal noise are restricted to waves normalized to complex reference impedances with *positive* real part. This study resulted in a generalized expression for matrix $\mathbf{C}_S$ that holds for any set of reference impedances, including complex values with *negative* real part.
For very high frequencies and low temperatures, an extension to the Nyquist theorem is required. All above thermal noise relations must be multiplied [710,720] with the Planck factor p(f,T) being:

$$p(f,T) = \frac{(hf/kT)}{\exp(hf/kT)-1} \qquad h = 6.626176\cdot 10^{-34}\ \text{[J.s]} = \text{Planck constant}$$

At room temperature this correction becomes significant for frequencies higher than 6000 GHz. Therefore, this correction may be omitted for many applications.

History
Thermal noise relations for $\mathbf{C}_Y$ and $\mathbf{C}_Z$ are known since 1955 by Twiss [711], as generalization of the Nyquist theorem [702]. Haus and Adler [716] derived in 1959 the same results. The eigenvalues of this matrix are invariant to lossless transformations that preserve the number of ports.
The relation for $\mathbf{C}_S$ for passive two-ports has been discussed in English[15] as early as 1959 by Gamo [717]. Several years later, Bosma [719] discussed similar topics[16] .
An alternative way of proving the thermal noise relations for $\mathbf{C}_S$ is discussed in [725]

7.2.6. Conclusions

Various matrix methods have been summarized to represent noisy linear networks as blackbox. The review was focused on a simultaneous discussion of using z-, y- and s-parameters, without any restrictions on the reference impedance Z_N. It resulted in generalized Norton- Thévenin- and wave representations of (noise) sources.

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15 Gamo refers to a Japanese publication from Takahasi (1952) on a generalization of the Nyquist theorem. This might be the first publication on this topic.

16 We observed that many authors referred to Bosma as the first who derived an expression for $\mathbf{C}_S$. Some of them identify this expression as Bosma's theorem. Historically, there are no grounds for this designation.

The use of correlation matrices has many advantages. They facilitate transformation between various representation formats with ease, their dimension is simple to reduce, they facilitate a generalized description of thermal noise and they facilitate the simulation of arbitrary noisy circuits. The simplicity of implementing circuit simulator dealing with noise has been discussed.
Although these methods are essentially known from the literature, many articles and textbook discussions have contributed to produce this detailed overview. This is because many textbook discussions are focused on isolated topics or discus methods that are restricted to real reference impedances or to positive complex reference impedances. Note that some authors use different definitions.

7.3. Two-port noise parameters

An important group of multi-port networks are two-ports. Automated measurement setups, dedicated to full two-port noise measurements at microwave frequencies, have come to the market. Moreover, transistor manufacturers are (sparingly) providing measured two-port noise parameters of their devices, to complement measured transfer parameters. These developments enable the application of two-port noise parameters as integral part of a design strategy for wideband amplifiers.
One can distinct different groups of noise parameters, as is summarized below:

- Designers of low-noise microwave circuits are using noise parameters for years. Their design goal is minimizing the noise factor, as explained in section 7.3.2, which can successfully be realized within small[17] frequency bands. *Spot noise parameters* are dedicated to these narrow-band design goals, and therefore commonly used.
- Spot noise parameters are inconvenient when designing wideband amplifiers. *Matrix noise parameters* are more appropriated and have the additional advantage that the underlying concepts are also applicable to multi-port circuits.
- Designers of wideband amplifiers preferably use equivalent circuit models, extracted from noise measurements. This study resulted in the development of *autonomous noise parameters* that simplify the extraction of adequate noise models for synthesis purposes.

This section 7.3 discusses all these noise parameters to facilitate input noise calculations, parameter conversion and parameter measurement. The first two groups are essentially known concepts, although this study generalized the validation of the spot noise parameters. Autonomous noise parameters are promising new concepts.

7.3.1. Matrix noise parameters, dedicated to two-ports

All previously defined Norton-, Thévenin- and wave representations in matrix format are applicable to representing two-port noisy networks. Additionally, many other matrix representations are in use.
Using multiple noise representations can sometimes lead to more efficient analysis, as has been demonstrated by Hillbrandt and Russer [721]. Additionally, Hartmann [722] distinct twelve matrix representations dedicated to two-port networks. Figure 7.5 shows various representations of two-ports with two equivalent sources, each associated with a preferred matrix format.

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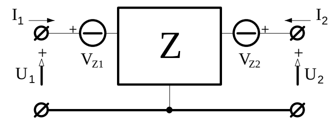
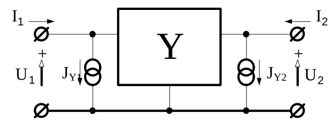
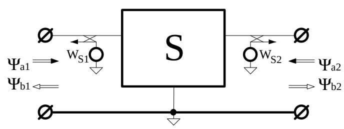
[17] The word 'small' is related to the center frequency of the frequency band of interest. Ten percent of the center frequency f_c is relatively small, although this is more than 1 GHz bandwidth at f_c=10GHz center frequency. The word 'spot' is used to emphasize that the application of spot noise parameters is primarily focussed on single frequencies.



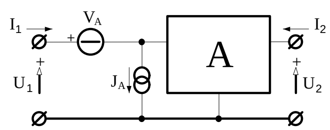
$$\mathbf{U} = \mathbf{Z}\cdot\mathbf{I} + \mathbf{X}_Z$$
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\cdot\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} V_{Z1} \\ V_{Z2} \end{bmatrix}$$



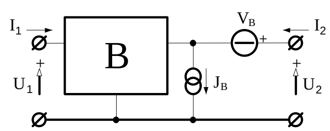
$$\mathbf{I} = \mathbf{Y}\cdot\mathbf{U} + \mathbf{X}_Y$$
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} J_{Y1} \\ J_{Y2} \end{bmatrix}$$



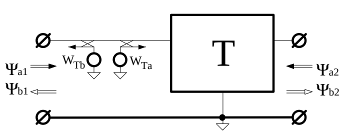
$$\boldsymbol{\Psi}_b = \mathbf{S}\cdot\boldsymbol{\Psi}_a + \mathbf{X}_S$$
$$\begin{bmatrix} \Psi_{b1} \\ \Psi_{b2} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\cdot\begin{bmatrix} \Psi_{a1} \\ \Psi_{a2} \end{bmatrix} + \begin{bmatrix} W_{S1} \\ W_{S2} \end{bmatrix}$$



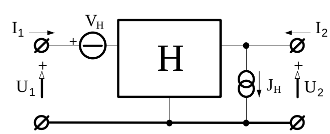
$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\cdot\begin{bmatrix} +U_2 \\ -I_2 \end{bmatrix} + \begin{bmatrix} V_A \\ J_A \end{bmatrix}$$



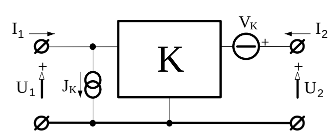
$$\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\cdot\begin{bmatrix} +U_1 \\ -I_1 \end{bmatrix} + \begin{bmatrix} V_B \\ J_B \end{bmatrix}$$



$$\begin{bmatrix} \Psi_{a1} \\ \Psi_{b1} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}\cdot\begin{bmatrix} \Psi_{b2} \\ \Psi_{a2} \end{bmatrix} + \begin{bmatrix} W_{Ta} \\ W_{Tb} \end{bmatrix}$$



$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\cdot\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} V_H \\ J_H \end{bmatrix}$$



$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} J_K \\ V_K \end{bmatrix}$$

Fig 7.5 *Reepresentations of internal (noise) sources of two-ports with two equivalent sources. The overall calculation effort is simplified when the associated matrix representation is used.*

Equivalent input noise correlation matrices

The most important two-port noise representations are those that have all equivalent (noise) sources transformed to the input port. They are preferably associated with chain parameters (A-matrix) or transmission parameters (T-matrix) to harmonize noise and transfer calculations. Their definition is:

$$\mathbf{C}_A \overset{\text{def}}{=} \begin{bmatrix} \langle V_A \cdot V'_A \rangle & \langle V_A \cdot J'_A \rangle \\ \langle J_A \cdot V'_A \rangle & \langle J_A \cdot J'_A \rangle \end{bmatrix} = \begin{bmatrix} S_V & S_{V|J} \\ S_{J|V} & S_J \end{bmatrix} \qquad \textit{for voltages and currents}$$

$$\mathbf{C}_T \overset{\text{def}}{=} \begin{bmatrix} \langle W_A \cdot W'_A \rangle & \langle W_A \cdot W'_B \rangle \\ \langle W_B \cdot W'_A \rangle & \langle W_B \cdot W'_B \rangle \end{bmatrix} = \begin{bmatrix} S_a & S_{a|b} \\ S_{b|a} & S_b \end{bmatrix} \qquad \textit{for waves}$$

These input noise parameters are measurable, independently from measurements of two-port transfer parameters. Specifying device noise as $\mathbf{C}_A$ or $\mathbf{C}_T$, enables the analysis of optimal noise performance without any knowledge on device transfer. $\mathbf{C}_A$ and $\mathbf{C}_T$ are input noise parameters among many others, see section 7.3.2. Nevertheless, these parameters yield in the most regular expressions for analyzing equivalent input noise.

Two-port transformation rules for correlation matrices

In section 7.2.1 various transformation rules are discussed for equivalent sources in multi-port networks. These transformation rules have ($\mathbf{X}_B = \mathbf{\Phi}_{BA} \cdot \mathbf{X}_A$) as general form. This regular form is maintained when the matrix representations of figure 7.5 are used. The associated two-port transformation matrices $\mathbf{\Phi}$ are summarized in figure 7.6.

$\mathbf{X}_B=\mathbf{\Phi}_{BA}\cdot\mathbf{X}_A$	$\mathbf{X}_Y$	$\mathbf{X}_Z$	$\mathbf{X}_A$	$\mathbf{X}_S$	$\mathbf{X}_T$
$\mathbf{\Phi}_{Y\otimes} =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -y_{11} & -y_{12} \\ -y_{21} & -y_{22} \end{bmatrix}$	$\begin{bmatrix} -y_{11} & 1 \\ -y_{21} & 0 \end{bmatrix}$	$(2\cdot\mathbf{r}_N\cdot\mathbf{Y})/(\mathbf{1}-\mathbf{S})$	
$\mathbf{\Phi}_{Z\otimes} =$	$\begin{bmatrix} -z_{11} & -z_{12} \\ -z_{21} & -z_{22} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -z_{11} \\ 0 & -z_{21} \end{bmatrix}$	$(2\cdot\mathbf{r}_N)/(\mathbf{1}-\mathbf{S})$	
$\mathbf{\Phi}_{A\otimes} =$	$\begin{bmatrix} 0 & a_{12} \\ 1 & a_{22} \end{bmatrix}$	$\begin{bmatrix} 1 & -a_{11} \\ 0 & -a_{21} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		
$\mathbf{\Phi}_{S\otimes} =$	$(\mathbf{1}-\mathbf{S})/(2\cdot\mathbf{r}_N\cdot\mathbf{Y})$	$(\mathbf{1}-\mathbf{S})/(2\cdot\mathbf{r}_N)$		$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -s_{11} & 1 \\ -s_{21} & 0 \end{bmatrix}$
$\mathbf{\Phi}_{T\otimes} =$				$\begin{bmatrix} 0 & -t_{11} \\ 1 & -t_{21} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Fig 7.6 *Matrices to transform the sources of figure 7.5 from format* $\mathbf{X}_A$ *to format* $\mathbf{X}_B$*. For the definition of* $\mathbf{r}_N$*, see section 2.2.1. The transformation matrices using s-parameters hold for waves normalized to arbitrary reference impedances* Z_n

Hillbrandt and Russer [721] discussed this regularity for Y-, Z- and A-matrices to demonstrate how matrix noise parameters simplify two-port noise analysis. When cascading two-ports, specified in **A**- or **T**- matrices, the input noise correlation matrices become (assuming equal reference impedance Z_0):

$$\mathbf{C}_A = \mathbf{C}_{A1} + \mathbf{A}_1 \cdot \mathbf{C}_{A2} \cdot \mathbf{A'}_1$$
$$\mathbf{C}_T = \mathbf{C}_{T1} + \mathbf{T}_1 \cdot \mathbf{C}_{T2} \cdot \mathbf{T'}_1$$

Equivalent input excess noise of two-ports

Figure 7.7a and 7.7b show two equivalent noise models representing the noise of two-ports observed from their input port. These are (partly) correlated sources.

To simplify manual circuit analysis using partly correlated noise sources, it is convenient to replace them by sources that are fully correlated and by other sources that are fully uncorrelated. Figure 7.7c, -d and -e illustrate this concept in various representations. The correlation factors Y_c, Z_c and Γ_c represent the complex ratio between two fully correlated sources, as illustrated in the associated correlation matrices $\mathbf{C}_A$ and $\mathbf{C}_T$.

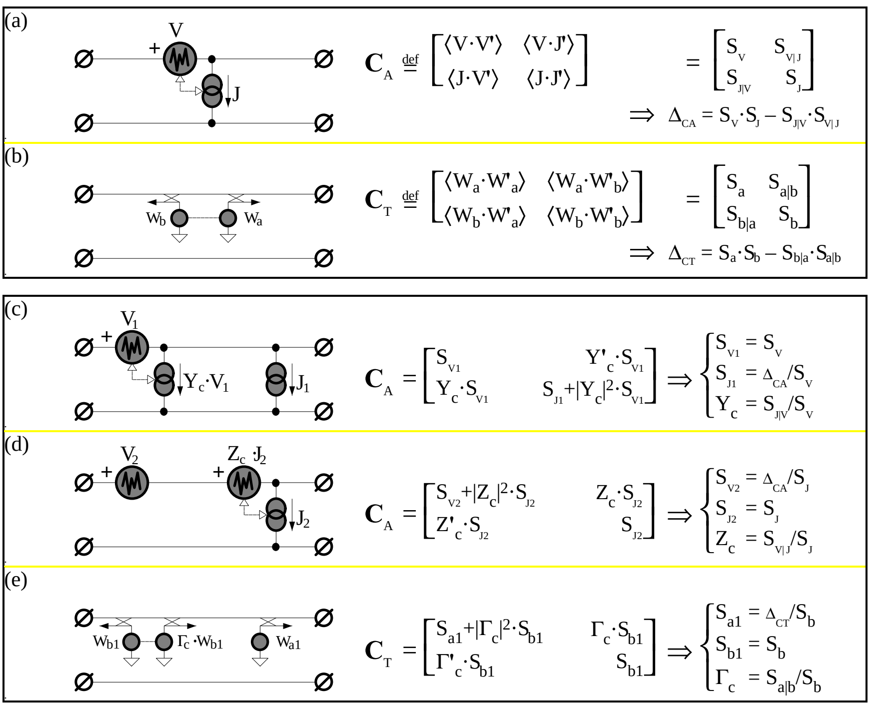


Fig 7.7 *Two (partly) correlated noise sources can be represented by two fully correlated sources and an additional uncorrelated noise source. This approach simplifies manual circuit analyses of input noise.*

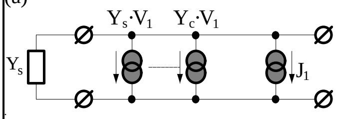
An important application of the models in figure 7.7 is the calculation of the input excess-noise level when the noisy two-port input is connected with a source impedance. Excess noise is the device noise in *excess* to the thermal noise of the source impedances. It equals the total input noise when the source impedance is cooled down to temperature T=0.

Figure 7.8 quantifies the input excess noise as current, voltage or wave, in various ways. This excess noise is the combination of all equivalent device noise sources transformed to the input port. When the total input noise is requested, add the thermal noise of the source impedance. This is one of:

$S_{is}=4kT\cdot re\{Y_s\}$ = thermal noise current
$S_{us}=4kT\cdot re\{Z_s\}$ = thermal noise voltage
$S_{ws}=kT\cdot(1-|\Gamma_s|^2)$ = thermal noise wave

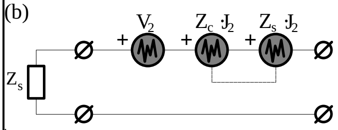
Excess noise levels

(a)



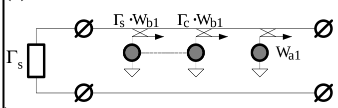
$S_i = |Y_s+Y_c|^2\cdot S_{V1} + S_{J1}$
$S_i = S_J + |Y_s|^2\cdot S_V + Y_s\cdot S_{V|J} + Y'_s\cdot S_{J|V}$
$S_i = S_J + |Y_s|^2\cdot S_V + 2\cdot re\{Y_s\cdot S'_{J|V}\}$
$S_i = S_J + |Y_s|^2\cdot S_V + 2\cdot re\{Y_s\}\cdot re\{S_{J|V}\} + 2\cdot im\{Y_s\}\cdot im\{S_{J|V}\}$
$S_i = [Y_s\,,\ 1] \cdot \mathbf{C}_A \cdot [Y_s\,,\ 1]'$

(b)



$S_u = |Z_s+Z_c|^2\cdot S_{J2} + S_{V2}$
$S_u = S_V + |Z_s|^2\cdot S_J + Z_s\cdot S_{J|V} + Z'_s\cdot S_{V|J}$
$S_u = S_V + |Z_s|^2\cdot S_J + 2\cdot re\{Z_s\cdot S'_{V|J}\}$
$S_u = S_V + |Z_s|^2\cdot S_J + 2\cdot re\{Z_s\}\cdot re\{S_{V|J}\} + 2\cdot im\{Z_s\}\cdot im\{S_{V|J}\}$
$S_u = [1\,,\ Z_s] \cdot \mathbf{C}_A \cdot [1\,,\ Z_s]'$

(c)



$S_w = |\Gamma_s+\Gamma_c|^2\cdot S_{b1} + S_{a1}$
$S_w = S_a + |\Gamma_s|^2\cdot S_b + \Gamma_s\cdot S_{b|a} + \Gamma'_s\cdot S_{a|b}$
$S_w = S_a + |\Gamma_s|^2\cdot S_b + 2\cdot re\{\Gamma_s\cdot S'_{a|b}\}$
$S_w = S_a + |\Gamma_s|^2\cdot S_b + 2\cdot re\{\Gamma_s\}\cdot re\{S_{a|b}\} + 2\cdot im\{\Gamma_s\}\cdot im\{S_{a|b}\}$
$S_w = [1\,,\ \Gamma_s] \cdot \mathbf{C}_T \cdot [1\,,\ \Gamma_s]'$

Fig 7.8 *Various related expressions for evaluating the device excess noise at specified source impedance. This excess noise equals the total equivalent input noise when the source impedances is cooled down to T=0 Kelvin.*

Calculations of equivalent input noise using voltages and currents are known since 1955 by Rothe and Dahlke [712,713]. Additionally, they developed a method to represent the correlation with correlation impedances or -admittances as (positive and negative) circuit elements. This approach is not discussed here.
Wave methods have been discussed in the same period by Bauer and Rothe [714]. Penfield [718] proposed in 1962 to use noise waves, normalized to complex reference impedances, as will be discussed in the succeeding section 7.3.2. When Z_N is properly chosen ($Z_N=Z_{opt}$ see next section), the two noise waves become uncorrelated. This simplifies the noise wave analysis.

7.3.2. Conventional spot noise parameters for two-ports

The two-port noise-parameters discussed in section 7.3.1 originated from matrix concepts that are generally applicable to arbitrary multi-ports. This explains the regular nature of various expressions when using matrix noise parameters.
In the line of historical developments, other sets of noise parameters have been evolved. They have in common that they are associated with noise factor measurements as a function of source impedance. Those (spot) noise parameters are useful when designing

amplifier configurations with minimum noise factor, however, these design goals are not useful[18] for wideband amplifiers.
To our opinion, conventional spot noise parameters are inconvenient for designing low-noise wideband feedback amplifiers. Nevertheless, it has been common practice to specify transistor noise parameters in this way. Furthermore, many publications relate their results to spot noise parameters. Therefore they are discussed in this section. Furthermore, conversion rules are provided between spot noise parameters and matrix noise parameters.

The basic concepts of the *noise factor* or *noise figure* were introduced by Burgess, Friss and North [705,706,707] in the 1940's and later adopted by IRE standards [817,818,821]. It is a quantity that relates the amplifier noise to the thermal noise of the associated source impedance. The factor F by which the total equivalent input noise is greater than the thermal noise of the source impedance is defined as the noise factor.
When S_i and S_u represent the spectral intensity of equivalent excess noise currents and voltages, and Y_s and Z_s are the source admittance and -impedance, then F is defined as:

$$F \stackrel{\text{def}}{=} \begin{cases} = \dfrac{S_i + 4kT\cdot re\{Y_s\}}{4kT\cdot re\{Y_s\}} \\ = \dfrac{S_u + 4kT\cdot re\{Z_s\}}{4kT\cdot re\{Z_s\}} \end{cases} \qquad \text{definitions are restricted for } re\{Z_S\}>0$$

F-1 = *excess noise figure*

The noise factor F is a quantity that changes with the source impedance, and therefore its value is meaningless when this source impedance is not specified. Goldberg [708] has shown in 1948 a possibility of noise factor reduction by mismatching impedances (noise tuning), an observation that was theoretically supported by Rothe and Dahlke [713]. In 1960 IRE standards recommended [818,819] two-port noise measurements, for extracting the minimum noise factor F_{min} and the associated optimal source impedance (Z_{opt},Y_{opt} or Γ_{opt}). An additional parameter was required to facilitate the prediction of F for all source impedances (Z_s,Y_s or Γ_s) of interest. Initially an arbitrary additional parameter was chosen. A commonly used choice is (R_u or G_i) representing being an equivalent thermal noise resistance or conductance of noise voltage V or -current J as shown in figure 7.7a.
In 1967, Lange [822] proposed a more fundamental parameter N. This is still an arbitrary parameter, however, it has the property that its value is invariant under lossless transformation (as holds for F_{min} too). This parameters is especially advantageous when the terminals of an intrinsic device are not directly accessible, e.g. due to the packaging of a chip.
The relations between matrix noise parameters and all these conventional spot noise parameters are shown in figure 7.9 and 7.10.

[18] The noise figure is minimal for a specific 'optimal' source impedance. This requires an impedance transformation of the source impedance with a loss-free 'matching' network. This condition can easily be realized within a small frequency band using resonating circuitry such as a stub. On the other hand, minimum noise figure cannot be achieved for all frequencies within a wide frequency band.

$\mathbf{C}_A$	$\mathbf{C}_T$
$N \stackrel{def}{=} \frac{\sqrt{c_{A11}\cdot c_{A22} - im\{c_{A12}\}^2}}{4\cdot k\cdot T}$ $F_{min} = 1 + 2\cdot N + \frac{re\{c_{A12}\}}{2\cdot kT}$ $Y_{opt} = \frac{4kT\cdot N + j\cdot im\{c_{A12}\}}{c_{A11}}$	$N = \frac{\sqrt{(c_{T11}+c_{T22})^2 - 4\cdot\lvert c_{T12}\rvert^2}}{4\cdot k\cdot T}$ $F_{min} = 1 + 2\cdot N + \frac{\lvert c_{T11} - c_{T22}\rvert}{2\cdot kT}$ $\Gamma_{opt} = \frac{2\cdot c_{T12}}{\sigma\cdot 4kT\cdot N + c_{T11} + c_{T22}}$

$$R_u \stackrel{def}{=} \frac{c_{A11}}{4kT} = \frac{N}{re\{Y_{opt}\}} = \frac{N}{re\{Z_0\}}\cdot\frac{|Z_0+Z'_0\cdot\Gamma_{opt}|^2}{1-|\Gamma_{opt}|^2}$$

$$G_i \stackrel{def}{=} \frac{c_{A22}}{4kT} = \frac{N}{re\{Z_{opt}\}} = \frac{N}{re\{Z_0\}}\cdot\frac{|1-\Gamma_{opt}|^2}{1-|\Gamma_{opt}|^2}$$

$$Z_{opt} = 1/Y_{opt} = \frac{Z_0 + Z'_0\cdot\Gamma_{opt}}{1-\Gamma_{opt}}$$

$$Y_{opt} = 1/Z_{opt} = \frac{1-\Gamma_{opt}}{Z_0 + Z'_0\cdot\Gamma_{opt}}$$

$$\Gamma_{opt} = \frac{Z_{opt} - Z_0}{Z_{opt} + Z'_0} = \frac{1 - Y_{opt}\cdot Z_0}{1 + Y_{opt}\cdot Z'_0}$$

Fig 7.9 *Definitions of various spot noise parameters, normalized to arbitrary reference impedance (including negative complex values, differently for each port).*

$$\mathbf{C}_A = \begin{bmatrix} c_{A11} & c_{A12} \\ c_{A21} & c_{A22} \end{bmatrix} = kT\cdot(F_{min}-1)\cdot\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} + \frac{4kT\cdot N}{re\{Y_{opt}\}}\cdot\begin{bmatrix} 1 \\ -Y_{opt} \end{bmatrix}\cdot\begin{bmatrix} 1 \\ -Y_{opt} \end{bmatrix}^{\prime}$$

$$\mathbf{C}_A = \begin{bmatrix} c_{A11} & c_{A12} \\ c_{A21} & c_{A22} \end{bmatrix} = kT\cdot(F_{min}-1)\cdot\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} + \frac{4kT\cdot N}{re\{Z_{opt}\}}\cdot\begin{bmatrix} -Z_{opt} \\ 1 \end{bmatrix}\cdot\begin{bmatrix} -Z_{opt} \\ 1 \end{bmatrix}^{\prime}$$

$$\mathbf{C}_T = \begin{bmatrix} c_{T11} & c_{T12} \\ c_{T21} & c_{T22} \end{bmatrix} = \sigma\cdot kT\cdot(F_{min}-1)\cdot\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \frac{\sigma\cdot 4kT\cdot N}{1-|\Gamma_{opt}|^2}\cdot\begin{bmatrix} +\Gamma_{opt} \\ 1 \end{bmatrix}\cdot\begin{bmatrix} +\Gamma_{opt} \\ 1 \end{bmatrix}^{\prime}$$

$$F(Y_s) = F_{min} + \frac{N}{re\{Y_{opt}\}}\cdot\frac{|Y_s-Y_{opt}|^2}{re\{Y_s\}} = F_{min} + R_u\cdot\frac{|Y_s-Y_{opt}|^2}{re\{Y_s\}}$$

$$F(Z_s) = F_{min} + \frac{N}{re\{Z_{opt}\}}\cdot\frac{|Z_s-Z_{opt}|^2}{re\{Z_s\}} = F_{min} + G_i\cdot\frac{|Z_s-Z_{opt}|^2}{re\{Z_s\}}$$

$$F(\Gamma_s) = F_{min} + \frac{4\cdot N}{1-|\Gamma_{opt}|^2}\cdot\frac{|\Gamma_s-\Gamma_{opt}|^2}{1-|\Gamma_s|^2} = F_{min} + \frac{4\cdot R_u/Z_0}{|1+\Gamma_{opt}|^2}\cdot\frac{|\Gamma_s-\Gamma_{opt}|^2}{1-|\Gamma_s|^2}$$

$$F(Y_s) = 1 + \frac{1}{4kT\cdot re\{Y_s\}}\cdot\left[+Y_s\,,\ 1\right]\cdot\mathbf{C}_A\cdot\left[+Y_s\,,\ 1\right]^{\prime}$$

$$F(Z_s) = 1 + \frac{1}{4kT\cdot re\{Z_s\}}\cdot\left[1\,,\ +Z_s\right]\cdot\mathbf{C}_A\cdot\left[1\,,\ +Z_s\right]^{\prime}$$

$$F(\Gamma_s) = 1 + \frac{1}{kT\cdot|(1-|\Gamma_s|^2)|}\cdot\left[1\,,\ -\Gamma_s\right]\cdot\mathbf{C}_T\cdot\left[1\,,\ -\Gamma_s\right]^{\prime}$$

Fig 7.10 *Relations between spot noise parameters and matrix noise parameters and noise figure. The validation of these relations are restricted to* $re\{Y_s\}>0$, Penfield [718] proposed in 1962 to use waves normalized to Z_{opt} as reference impedance. Such a choice simplifies various relations since Γ_{opt} becomes zero, matrix parameter c_{T12} becomes zero, and the noise waves in figure 7.7b become uncorrelated. This can easily be verified from the relations in figure 7.9.

A general accepted way of specifying spot noise parameters is lacking, although an increasing number of manufacturers specify their semiconductor devices in terms of: [F_{min}, $re\{\Gamma_{opt}\}$, $im\{\Gamma_{opt}\}$, R_i/Z_0]. Other manufacturers specify in terms of noise circles, which requires graphical extraction of the spot noise parameters from these plots.
Some authors use the factor (4·N) as noise parameter instead of (N), while referring to the original definition of Lange [822]. Other authors use the *noise measure* M, defined by Haus and Adler [715,716], as one of the spot noise parameters. This illustrates how confusing it is to intercompare the noise of various devices.

7.3.3. Autonomous noise parameters for two-ports.

So far, two groups of two-port noise parameters have been discussed. The matrix- as well as the spot noise parameters are fully capable of representing measured results on arbitrary noisy two-ports. This approach is effective for computer assisted circuit analysis, however, it is inconvenient for circuit synthesis[19].
The aim of section 7.3.3 is to find a parameter representation of noisy two-ports, using (1) uncorrelated virtual noise sources characterized by (2) simple spectra over the full frequency band of interest. This might simplify the synthesis of noisy circuits and the development of transistor noise models, similarly to the concept of two-port *virtual circuit parameters* as introduced in section 2.3.2. We refer to the intensity spectra of these virtual noise sources as *autonomous noise parameters*[20].

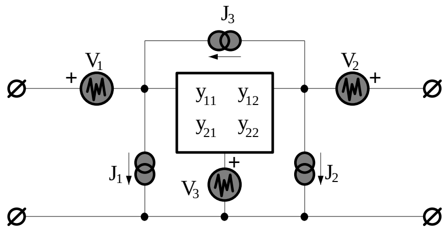


Fig 7.11 *Autonomous noise parameters represent the intensity spectra of the uncorrelated noise sources in this circuit.*

Autonomous noise parameters are related with a chosen topology, for instance the topology in figure 7.11. This topology is characterized by the fact that all virtual noise sources are extended to the output nodes of an inner noiseless blackbox. When the noise levels of two virtual noise sources are given (or chosen) the levels of the remaining four sources are unique.

19 The purpose of analysis is to find the properties of a circuit when it has been designed. Circuit simulations as well as measurements are adequate methods. The purpose of synthesis is to find a circuit that meets a set of pre-defined properties. This is the reverse problem.

20 The term *autonomous* is chosen to emphasize that the virtual noise sources are to be *uncorrelated*.

The topology of figure 7.11 is not the only one to be considered, although other topologies have not been investigated. Alternative topologies can be based on a virtual circuit representation of a noiseless blackbox, that include internal (virtual) nodes. This facilitates the extension of virtual noise sources, connected with external as well as internal virtual nodes. Finding the most convenient virtual circuit topology for a device of interest is a matter of trial and error in practice.

Extraction algorithm.

All six noise sources of the topology in figure 7.11 are uncorrelated by definition, and the two-port transfer parameters are assumed to be known. Six noise levels are to be extracted from four (matrix) noise parameters, and therefore a unique solution does not exist. When the values for two of the six are chosen, for instance $S_{V3}=0$ and $S_{J3}=0$, then the remaining values can be extracted uniquely from measured matrix noise parameters. Let $\mathbf{C}_Y$ be the current correlation matrix of a two-port, and let $\mathbf{Y}$ be the associated two-port admittance parameters. To evaluate $\mathbf{C}_Y$ the noise currents can be inserted as discussed for circuit simulators in section 7.2.4. The noise voltages can be inserted using similar techniques and using the voltage to current transformation rule as summarized in figure 7.6. The extracted parameters $[S_{J1}, S_{J2}, S_{V1}, S_{V2}]$ from the measured $\mathbf{C}_Y$ and the chosen values $[S_{J3}, S_{V3}]$ are:

$$\mathbf{C}_Y = S_{J1}\cdot\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + S_{J2}\cdot\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} + S_{J3}\cdot\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + S_{V1}\cdot\mathbf{Y}\cdot\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\cdot\mathbf{Y'} + S_{V2}\cdot\mathbf{Y}\cdot\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\cdot\mathbf{Y'} + S_{V3}\cdot\mathbf{Y}\cdot\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\cdot\mathbf{Y'}$$

$$\begin{bmatrix}c_{Y11} & -1 & -(y_{11}+y_{12})\cdot(y'_{11}+y'_{12})\\ c_{Y21} & 1 & -(y_{21}+y_{22})\cdot(y'_{11}+y'_{12})\\ c_{Y12} & 1 & -(y_{11}+y_{12})\cdot(y'_{21}+y'_{22})\\ c_{Y22} & -1 & -(y_{21}+y_{22})\cdot(y'_{21}+y'_{22})\end{bmatrix}\cdot\begin{bmatrix}1\\S_{J3}\\S_{V3}\end{bmatrix} = \begin{bmatrix}1 & 0 & y_{11}\cdot y'_{11} & y_{12}\cdot y'_{12}\\ 0 & 0 & y_{21}\cdot y'_{11} & y_{22}\cdot y'_{12}\\ 0 & 0 & y_{11}\cdot y'_{21} & y_{12}\cdot y'_{22}\\ 0 & 1 & y_{21}\cdot y'_{21} & y_{22}\cdot y'_{22}\end{bmatrix}\begin{bmatrix}S_{J1}\\S_{J2}\\S_{V1}\\S_{V2}\end{bmatrix}$$

$$\boxed{\begin{bmatrix}S_{J1}\\S_{J2}\\S_{V1}\\S_{V2}\end{bmatrix} = \begin{bmatrix}1 & 0 & y_{11}\cdot y'_{11} & y_{12}\cdot y'_{12}\\ 0 & 0 & y_{21}\cdot y'_{11} & y_{22}\cdot y'_{12}\\ 0 & 0 & y_{11}\cdot y'_{21} & y_{12}\cdot y'_{22}\\ 0 & 1 & y_{21}\cdot y'_{21} & y_{22}\cdot y'_{22}\end{bmatrix} \backslash \begin{bmatrix}c_{Y11} & -1 & -(y_{11}+y_{12})\cdot(y'_{11}+y'_{12})\\ c_{Y21} & 1 & -(y_{21}+y_{22})\cdot(y'_{11}+y'_{12})\\ c_{Y12} & 1 & -(y_{11}+y_{12})\cdot(y'_{21}+y'_{22})\\ c_{Y22} & -1 & -(y_{21}+y_{22})\cdot(y'_{21}+y'_{22})\end{bmatrix}\cdot\begin{bmatrix}1\\S_{J3}\\S_{V3}\end{bmatrix}}$$

This left-hand matrix division always results in a real and unique solution. On the other hand, it will not necessarily yield positive values. An improper choice of $[S_{J3}, S_{V3}]$ may therefore result in solutions that have no meaning at all. The development of simple criteria for choosing $[S_{J3}, S_{V3}]$ is of interest for further investigation.

Extraction in practice

It is expected that the topology in figure 7.11 is convenient for the representation of noise in transistors. This is because that topology is closely related to simple physical noise models based on uncorrelated white noise sources. These physical models are shown in figure 7.12, and have often proven adequate for the design of [406] wideband amplifiers. When required, voltage noise sources can be added in series with the device terminals to represent thermal noise effects caused by series resistance.

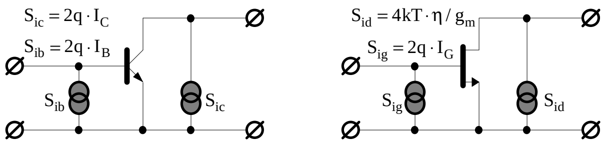


***Fig 7.12** Simple noise models for transistors. Most noise sources originate from shot noise associated with bias currents or leakage currents. The channel noise in the FET is represented acting as if the transconductance g_m is providing thermal noise. The correction factor* h *is a value close to one, to be extracted from measurements. The FET model ignores the influence of induces gate noise because its contribution is of minor importance when designing wideband amplifiers [406]*

Full two-port noise parameters of transistors are required to demonstrate the correctness of our proposition. In contrast to full two-port *transfer* parameters, these noise parameters are sparingly available and usually restricted to a few frequencies above 1 GHz. The lack of adequate measuring instruments has resulted in the development of an innovative setup for measuring two-port noise parameters, as described in section 8.5. Preliminary results [834], ranging from 10 to 500 MHz, indicate that the use of autonomous noise parameter is a promising technique. It is expected that this new approach simplifies the extraction of transistor noise models, similarly to the extraction of transistor transfer models from virtual circuit parameters

7.3.4. Conclusions

Two-port noise measurements can be specified in several different ways. Three classes of two-port noise parameters have been discussed in detail, as listed below:

- *Matrix noise parameters* are deduced from general applicable concepts. Various relations are therefore expressed in a regular way. This makes them most convenient for analyzing noise in wideband amplifiers.
- *Spot noise parameters* originate from historical developments on minimizing noise in very small frequency bands. Using these parameters in other applications result in unnecessary complicated expressions. Therefore they are inconvenient when designing wideband amplifiers.
- *Autonomous noise parameters* are promising new concepts, originated in this study. They may simplify the extraction of transistor noise models from two-port noise measurements. These parameters represent the spectral intensity of uncorrelated noise sources.

A general accepted way of specifying noise is lacking, although an increasing number of manufacturers is specifying their semiconductor devices in a corresponding spot noise parameter format. Most textbooks and publications are focused on some specific representation method. In some of them slightly different definitions on noise parameters are used.

This study resulted in generalized transformation rules between spot noise parameters and matrix noise parameters. They are extended to wave applications normalized to arbitrary reference impedance, including complex values with negative real part.